PERGAMON

Omega, Int. J. Mgmt Sci. 27 (1999) 241–258

omega
The International Journal of Management Science

Mutual fund performance appraisals: a multi-horizon perspective with endogenous benchmarking

Matthew R. Morey[a], Richard C. Morey[b,*]

[a]Department of Economics, Fordham University, Bronx, New York, NY 10458, USA
[b]School of Hotel Management and Tourism, Griffith University, Gold Coast Mail Center 50, Queensland 4218, Australia

Received 1 June 1997; accepted 1 June 1998

Abstract

With over 6500 mutual funds available to investors, industry data show that consumers pay a great deal of attention to the ratings of mutual funds. In spite of this attention, however, much controversy surrounds the various industry approaches to the rating of mutual funds. Many industry rating approaches use subjective weights to integrate fund performances over different time horizons; this can give rise to quite different ratings, depending upon the relative importances assigned to different horizons. In this paper, we present two basic quadratic programming approaches for identifying those funds that are strictly dominated, *regardless* of the weightings on the different time horizons being considered, relative to their mean returns and risks. This effort can be viewed as a novel application of the philosophy of data envelopment analysis, a relatively new, non-parametric frontier estimation technique which focuses on estimating 'radial' contraction/expansion potentials. These approaches eliminate any need for subjective tradeoffs, vis-á-vis the importance or meaningfulness of performances over the different horizons. Finally, much useful sensitivity information is automatically provided. Also, in contrast to many studies of mutual fund performance, our approaches *endogenously* determine a custom-tailored benchmark portfolio to which each mutual fund's performance is compared. All of our approaches are illustrated on a sample of twenty-six actual mutual funds. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Mutual funds; Benchmarking; Ratings; Data envelopment analysis; Multi-horizon; Risks; Quadratic programming

1. Introduction and motivation

With over 6500 mutual funds available to investors and over US$3.145 trillion in mutual funds (see Ref. [26]), increasing importance has been placed on the rankings and/or ratings of mutual funds. Numerous business magazines and private firms now specialize in giving regular, exhaustive rankings and ratings of mutual funds. Indeed, industry data shows that consumers pay a great amount of attention to these evaluations. For example, in a recent study [28], 97% of the money flowing into no-load equity funds between January and August 1995 was invested into funds which were rated as 5 star or 4 star funds (the top and next to top ratings) by Morningstar Inc., an industry leader in evaluating mutual funds; further funds with less than 3 stars actually suffered a net outflow of funds over the same period [28].

The typical investor, attempting to choose a mutual fund in which to invest, has available various funds' performances over: the past year to date; 1 year; 3 years; 5 years and 10 years. These various time-horizon performances are made available to potential investors largely because they provide considerable additional information over that given by a fund's performance over just a single time period. However, the investor is typically left to his own intuition and devices to determine which performances (i.e. over which horizons)

* Corresponding author. Tel.: +61-7-5594-8013; Fax: +61-7-5594-8507; E-mail: r.morey@bhm.gu.edu.au.

0305-0483/99/$ - see front matter
PII: S0305-0483(98)00043-7

are the most meaningful in terms of predicting future performance.

Let us review how two of the biggest mutual fund ranking services approach this controversial issue. Lipper Analytical Services breaks down the mutual fund universe into many small categories and then ranks funds for each category over different time horizons, based solely on returns; Lipper makes no adjustment for risk in their rankings, and they provide no single summary score for each fund. Rather, they simply rank each fund for each separate time horizon examined. This differs markedly from Morningstar's aggregated approach which arrives at a *single summary rating* (5 stars being the best, 1 star being the worst), combining all of the performances (and all of the risks) over three different horizons. Specifically, they first calculate a type of risk-adjusted return for the 3-year, 5-year and 10-year horizons; they then multiply these adjusted returns by weights of 0.2, 0.3 and 0.5 respectively. The resulting single number is then plotted along a bell curve to determine the fund's number of stars.

One of the problems with this latter method is that these weights are quite arbitrary. Indeed, Lipper has repeatedly stated that Morningstar's rankings are "subjective and can lead to mis-identification of high performing funds". For example, Lipper contends that in spite of a given mutual fund having a relatively poor recent performance, the fund may still be highly rated if its longer-term returns are strong [29]. This is due to the fact that Morningstar, Inc. puts relatively little weight on the most recent performance. Indeed, these contentions of subjectivity in their approach stormed into such a fury that they prompted Morningstar to retort that Lipper Analytical's own system is a subjective evaluation since the decision as to which category to classify a fund can greatly influence its relative ranking [29].

In this paper we present two basic, alternative mutual-fund rating approaches that overcome the above problems with the Lipper Analytical and Morningstar approaches and yet still integrate the returns and risks. Our two quadratic programming approaches can be characterized as benchmarking, efficiency frontier approaches that were motivated by data envelopment analysis (see Ref. [3]), a recent non-parametric, *linear programming* approach, widely cited in the Operations Research literature. As will be shown in the paper, our approaches have the following desirable features:

(i) They *simultaneously* address a fund's total risk and total return performances over different horizons, e.g. 1, 3, 5 and 10 years.

(ii) The two approaches are able alternatively to emphasize either risk or return, while always capturing the other.

(iii) The results *do not utilize or depend* on any subjective weights relative to meaningfulness or importance of the various performances over the different time horizons.

(iv) Each mutual fund is evaluated relative to a custom-tailored, *endogenously* created benchmark fund (i.e. a convex combination of actual funds, i.e. a 'fund of funds') operating over exactly the same time periods. The benchmark fund, an actually achievable fund, is typically different for each Fund being evaluated. Hence, some of the performance measurement problems arising from using one fixed benchmark, as described by Ross [23], Lehman and Modest [13] and Grinblatt and Titman [8] are ameliorated. This point will be discussed more in Section 2[1].

(v) The approaches are transparent, with the final ratings having a clear economic interpretation.

(vi) They are non-parametric.

(vii) They use readily available data.

(viii) They provide, for the underperforming funds, the levels of returns and risks needed for the inefficient fund to be on the efficiency frontier.

Although our approaches satisfy all of the above properties, it clearly is not a panacea and has some caveats which will be discussed. Such caveats notwithstanding, our approaches will be shown to provide a novel and objective rationale for integrating a given mutual Fund's relative performances over many time horizons, without the need for subjective weights. As such it provides some quantitative insights useful in identifying those mutual funds that undeniably were the relative underperformers. It also should be viewed as a novel application of the basic philosophy underlying data envelopment analysis and may inspire other such applications.

The remainder of the paper is organized as follows: Section 2 provides a literature review and positioning of our approaches; Section 3 gives the detailed formulations for estimating the simultaneous radial expansion/contraction levels; Section 4 deals with extensions to estimate additional 'slack' potential and for further

[1] One method of evaluating mutual funds is to examine how the fund performs relative to the capital market line (CML), which is simply a straight line linking the risk-free rate with the market portfolio. If the ex-post risk and return performance is better than the CML, the mutual fund is considered to be very good. Our method, which is consistent with the Morningstar '# of Stars' approach, is to examine a fund's performance relative to the other funds in the same investment category, rather than just determining which funds are above or below the CML. Moreover, since our method uses an endogenous benchmarking approach, there is no problem in choosing which market index to use a benchmark. When using the CML approach, different results can be obtained by using different market indexes.

discriminating among 'tied' funds; Section 5 is the data section and provides some numerical illustrations, using 26 actual mutual funds operating over the period 1985–1995. In Section 5, we go into some detail regarding the evaluation of one of the 26 funds and then compare, for all the 26 funds, the rankings from our approaches with those of Morningstar's; Section 6 contains an associated Lagrangian analyses, with Section 7 containing conclusions and caveats. Appendix A and Appendix B include the detailed co-variance data and a derivation of the Lagrangian result.

2. Review of the literature and positioning of our approaches

2.1. Previous work on mutual fund performance measures

In terms of methods of measuring the performance of mutual funds, the literature is quite large. Early seminal papers include Refs. [10, 11, 15, 24, 27]. The performance measures developed in these papers all are based on the capital asset pricing model (CAPM) and encompass some variation of a risk-adjusted return where either the standard deviation of the mutual fund or a measure of the mutual fund's market risk, Beta, is used to control for risk. The Jensen and Treynor measures are single-index measures of performance as they compare each fund manager with a single benchmark, i.e. the proxy used for market risk. For example, the Jensen measure, one of the most widely-used performance measures, is the intercept of a regression of the excess return of the evaluated mutual fund on the excess return of a benchmark portfolio.

Roll [22] has shown that single-index measures of performance are sensitive to the type of benchmark portfolio used. That is, the Beta when using the Standard and Poors Index is not the same as the Beta calculated when using the Dow-Jones index as the benchmark portfolio. These problems with using the correct benchmark portfolios plus concern over the testability of the CAPM has led to use of multi-index models that embody the arbitrage pricing theory (APT) developed by Ross [23]. Researchers recognized that managers hold a wide array of assets besides the large stocks that are contained in the Standard and Poors or Dow Jones indexes. For example, some funds hold only bonds, some only small company stocks, some only international stocks. To control for this situation, multiple index performance measures have been developed that use indexes that most closely match the risk choice of the fund manager being evaluated. Sharpe [25], Elton et al. [5, 6] use multi-index models with market indexes to measure portfolio performance while Lehman and Modest [13], Connor and Korajczyk [4] and Grinblat and Titman [9] use multi-index models that employ empirically estimated indexes.

Similar to Roll [22], Lehman and Modest [13] also examine the issue of benchmark sensitivity. They examine whether the CAPM and APT measures of mutual fund performance are sensitive to the type of benchmark used. In examining 130 mutual funds, they found that the rankings of the particular funds were heavily dependent upon the choice of CAPM and APT benchmarks. In light of these problems with benchmarking sensitivity, Grinblatt and Titman [9] have developed a mutual fund performance measure which does not require the use of a benchmark portfolio. Instead, they employ a performance measure which uses changes in mutual fund portfolio holdings. They show, using the approach, that mutual fund managers earned significantly positive risk-adjusted returns in the 1976–1985 period[2].

2.2. Positioning of our approach

As with most of the literature on mutual fund performance our approaches to the funds are based on a benchmark portfolio. However, our methods do not depend upon one preselected benchmark portfolio as in the case of the single-index model; rather, we arrive at a different 'custom-tailored' benchmark for each evaluated fund. Our approach is a benchmark, efficiency frontier method that takes its inspiration from data envelopment analysis (DEA), a relatively recent non-parametric linear programming approach now being widely applied, especially in the public sector. This analytical approach, first stated by Charnes et al. [3] and inspired by Farrell's work [7], is particularly well suited to multi-input, multi-output situations where there are, for example, no a priori tradeoffs on the relative importances of various types of outputs,

[2] One other approach that we do not cover review but has some relevance to our approach is stochastic dominance. This approach has been used in ranking mutual funds by Joy and Porter [12], Meyer [17] and Okunev [21]. In this approach the rankings/ratings of mutual funds are based on families of utility functions. Generally speaking, there are a series of progressively stronger assumptions about investor behavior used in the stochastic dominance approach to ranking mutual funds. These assumptions lead directly to first-, second- and third-order stochastic dominance. First-order assumes an investor prefers more return to less return. Second-order assumes that, in addition to first order stochastic dominance, investors are risk adverse. Lastly, third-order stochastic dominance adds to the two assumptions of first-and second-order dominance the assumption that investors have decreasing absolute risk aversion.

e.g. number of processed felony versus misdemeanor cases in courts (as in Ref. [14]), discharges of various types in hospitals (as in Ref. [18]) or profit versus market share (as in, for example, Ref. [1]). To accomplish the above DEA utilizes the concept of maximum 'radial' expansion/contraction potentials whereby the emphasis is, in one case, on estimating how much *all* of the outputs can be increased, with no increase in *any* of the inputs. The analogy of the situation faced in evaluating mutual fund performance over different horizons includes: (1) a multiplicity of 'inputs', i.e. the levels of total risk being appropriated over various horizons; (2) a multiplicity of outputs, i.e. the resulting mean rates of return for the different time horizons of interest, and (3) no a priori tradeoffs readily available regarding the relative importance or meaningfulness of, say, the 3 year performance versus the 10-year performance.

Fig. 1 summarizes the types of data needed to evaluate a given mutual fund and the general types of quantitative insights available from the DEA inspired approach. The key statistics are the individual mutual fund's mean returns, its appropriated levels of risk (as measured by the variance of the fund's returns) and the correlations between the mutual fund's returns and other funds returns, *all over each* of the separate time horizons being evaluated. The correlations are needed to capture the variability (i.e. variance) of the benchmark fund's returns, a composite of the other funds' actual returns. The key outputs (*from each of the two basic approaches*) are: a single numerical score for each fund; the other mutual funds used to benchmark the fund being evaluated and the changes in an under-performing fund's mean returns and risks that would have been necessary so as to position the fund on the efficiency frontier (in the mean return/risk space).

Other work on evaluating mutual funds through the usage of DEA-like methodology is quite sparse. Murthi et al. [20] use a single horizon model where DEA is used to account for transaction costs in the ranking of mutual funds. They develop a 'generalized Sharpe index' which is a ratio of the fund's mean return divided by a weighted sum of various transaction costs and the fund's standard deviation of the returns. Other than this paper, however, there is, to our knowledge, no other application to mutual fund performance measures which use the basic philosophy and spirit of DEA.

3. Mathematical formulations of our basic approaches

3.1. Overview and notation

We first present two different approaches for identifying the clearly dominated mutual funds, without resorting to require *any* subjective information. The two approaches (see Figure 2) represent different paths to the 'efficiency frontier' in the mean return/risk hyperspace. In one approach, the focus is on *simultaneously increasing mean returns*, over *all* of the time horizons of interest, while holding the level of total risk appropriated in each horizon to be no larger than that actually experienced. (Total risk includes both systematic and non-systematic risk and is measured by the variance of the returns). In the alternative

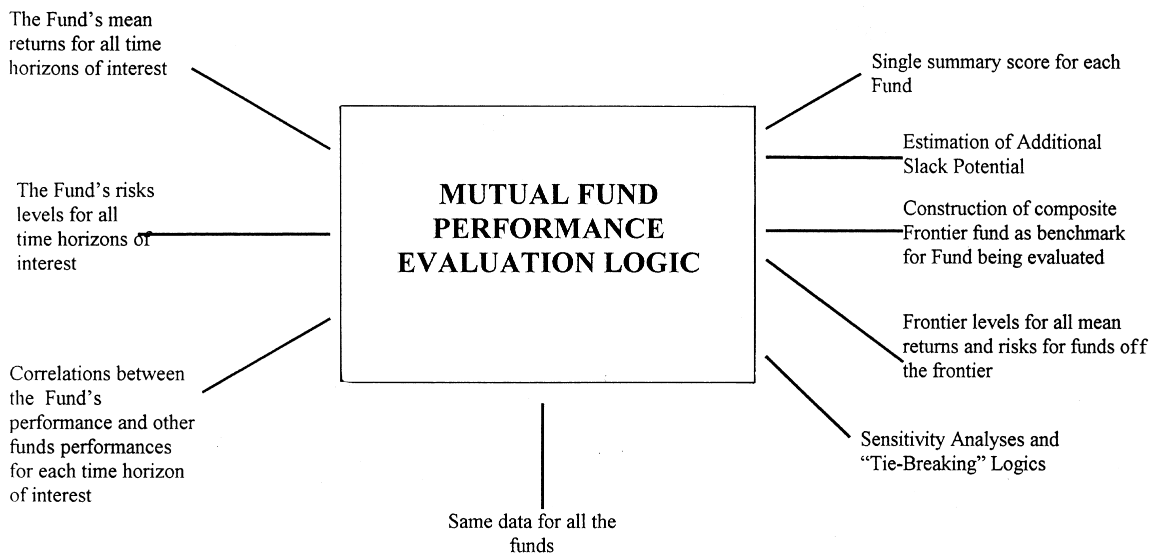

Fig. 1. Data requirements and general outputs from the evaluation logic applied to a given fund's performances.

approach, yielding a different endogenously created benchmarking fund, the thrust is on simultaneously reducing the levels of total risks, for *all* different time horizons, while insuring that none of the mean return levels are lowered. Both of the above 'radial' approaches neither require tradeoffs regarding the relative importances of different time horizons, nor even an ordering of the relative importances. However, to deal with the so called additional 'slacks' issue (addressed in Section 4), we shall use a pre-emptive programming approach in which a series of mathematical programs are solved sequentially to find out the truly maximum improvements possible and to help discriminate among 'tied' funds; this extension will require a simple ordering of the importances of the different horizons.

We shall use the following notation in the description of the various approaches:

Notation: Consider N mutual funds to be evaluated, indexed $j = 1, 2, \ldots, N$, where we suppose the particular focus, for the moment, is on the j_0th fund ($j_0 = 1, 2, \ldots, N$). That is, each of the following formulations will be solved N times, where j_0 varies from 1 to N. Let T denote the number of different time horizons of interest, indexed by $t = 1, 2, \ldots, T$. (As mentioned earlier, many mutual fund rating services focus on 3 distinct time horizons: 3, 5 and 10 years).

Finally, denote by $R_{j,t}$ ($j = 1, 2, \ldots, j_0, \ldots, N$; $t = 1, 2, \ldots, T$), a random variable whose realizations are, say the monthly percentage changes in the returns for the jth fund over the tth time horizon. Finally, let $E(R_{j,t})$ denote its mean; $\sigma^2_{j,t}$ ($j = 1, 2, \ldots, N$; $t = 1, 2, \ldots, T$) denotes its variance; $\mathrm{Cov}(R_{i,t}, R_{j,t})$ ($i = 1, 2, \ldots, j_0, \ldots, N$; $j = 1, 2, \ldots, N$; $t = 1, 2, \ldots, T$; $i \neq j$) denotes its covariances.

3.2. Mean return augmentation

Approach I (thrust is on simultaneously augmenting the mean rates of return for j_0th fund over all the horizons, with no increases in any of the total risks; this formulation will be referred to as the 'radial expansion formulation'):

Determine $w_j \geq 0$ ($j = 1, 2, \ldots, j_0, \ldots, N$) and $\theta \geq 1$ so that:

$$\sum_{j=1}^{N} w_j = 1 \tag{1}$$

$$\sum_{j=1}^{N} w_j^2 \sigma_{j,t}^2 + \sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} w_i w_j \,\mathrm{Cov}(R_{i,t}, R_{j,t}) \leq \sigma_{j_0,t}^2 \quad (t = 1, 2, \ldots, T) \tag{2}$$

$$\sum_{j=1}^{N} w_j E(R_{j,t}) \geq \theta E(R_{j_0,t}) \quad (t = 1, 2, \ldots, T) \tag{3}$$

$$\mathrm{Max}\ \theta \tag{4}$$

Consider Eqs. (1)–(4), a quadratic programming problem with decision variables, w_j and θ. The first constraint (Eq. (1)) is simply the convexity constraint which says that every dollar of the endogenously created, composite benchmarking frontier fund (for the j_0th fund) consists of w_j dollars of the jth fund. Such a fund was indeed realizable since in actuality one could have bought a 'fund of funds'. The second set of constraints (Eq. (2)) simply guarantees for each horizon that the resulting appropriated total risk level for the composite fund is no larger than that of the fund being evaluated; this constraint holds for all of the time horizons. Note that the left side of Eq. (2) is the variance of the random variable, $\Sigma_{j=1}^{N} w_j R_{j,t}$ and of course involves the appropriate covariances, it is exactly the same type of constraint as used by Markowitz [16] in his ground breaking single horizon effort. Moving to the constraints in Eq. (3), since $\theta \geq 1$, each constraint of this type requires that the composite fund have a mean rate of return for the tth time period which is *at least as large* as that experienced by the j_0th fund. The objective in Eq. (4) is to attempt to increase this simultaneous augmentation factor, θ, strictly above unity. If the resulting maximum θ (from Eqs. (1)–(4)) is strictly above one, then the j_0th fund is clearly dominated[3]. Note also that Eqs. (1)–(3) are always feasible since the solution $w_{j_0} = 1$, $\theta = 1$, always satisfies Eqs. (1)–(3). Hence the optimal θ can be seen to be at least one and hence of course there is no need to add it as a formal constraint.

In terms of a possible rating for the j_0th fund (vis-á-vis the other $N-1$ funds), subject to some non-trivial caveats to be subsequently discussed, one has the level of the resulting scalar, θ^*, i.e. the maximum *simultaneous* increase in the mean returns, with no increase

[3] We recognize that it may be possible to create a composite fund which increases the mean rate of return for some of the time horizons but not others. That is, if the above formulation, Eqs. (1)–(4), results in $\theta^* = 1$, the j_0th fund may still be truly inefficient in that it may be possible to increase a subset of the mean rates of return and not others. For example, we might be able to increase the 5-year return but not the 3-year and 10-year returns. The degree to which this could be accomplished could be ascertained by solving a second math programming formulation where the 'slacks' in Eq. (3) (for θ set equal to 1), could be maximized. This variation is covered in Section 4, together with its possible use in breaking 'ties'.

in any of the risks. A theta score of say 1.2 for a given fund signifies there is convex combination of other actual mutual funds for which its mean returns (over *all* horizons of interest) are *at least* 20% larger than that of the fund being evaluated, with no accompanying increases in *any* of the risk levels. Hence higher θ's are associated with poorer performance. The frontier funds will have an optimal theta value of one. However, even some of these funds may not be strictly efficient, based on consideration of its 'slacks' to be discussed in Section 4.

To summarize, if one is unable or unwilling to trade-off the relative meaningfulness of the resulting performances over different time horizons, then this approach for arriving at a single summary score has some appeal. In the spirit of classical economics' expansion/contraction paths, we are using the 'distance' from the frontier as our metric. However, one must use caution in using θ^* to rate mutual fund performances as the benchmark funds are being varied, and other 'slacks' have been ignored. These and other caveats are dealt with subsequently.

3.3. Risk contraction

Before getting into the details of 'slacks' (i.e. any improvement possible beyond the radial factor amount), consider the other 'radial' formulation, representing a different 'path' to the mean return/risk frontier (see Fig. 2 for the case of a single horizon).

Approach II (thrust is on *contracting* all of the risk levels, for all time horizons of interest for the j_0th fund, with no decrease in any of the mean returns, i.e. radial contraction):

$$\sum_{j=1}^{N} w_j = 1 \tag{5}$$

$$\sum_{j=1}^{N} w_j E(R_{j,t}) \geq E(R_{j_{0,t}}) \quad (t = 1, 2, \ldots, T) \tag{6}$$

$$\sum_{j=1}^{N} w_j^2 \sigma_{j,t}^2 + \sum_{\substack{i=1 \\ i \neq j}}^{N} \sum_{j=1}^{N} w_i w_j \operatorname{Cov}(R_{i,t}, R_{j,t}) \leq Z\sigma_{j_{0,t}}^2 \quad (t = 1, 2, \ldots, T) \tag{7}$$

$$\text{Min } Z \tag{8}$$

Note that Eqs. (5)–(7) is always feasible by taking $w_{j_0} = 1$, $Z = 1$ and that of course $Z^* \leq 1$.

It will also be demonstrated that the optimal values of Z and θ for a given fund are not simply the reciprocals of one another and that different orderings can result, based on whether one uses the scalar θ^* or Z^*.

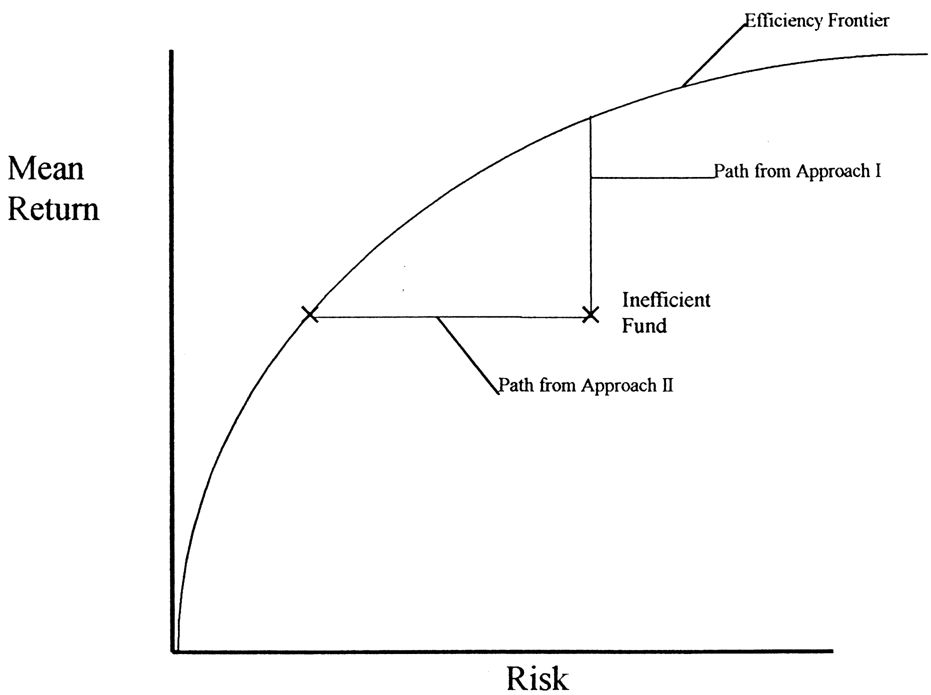

Fig. 2. Different paths to the efficiency frontier.

Further, the resulting two endogenously created benchmark funds will be quite different.

Both approaches, i.e. Eqs. (1)–(4) and Eqs. (5)–(8), yield as byproducts Lagrangians related to the constraints of Eqs. (1)–(3) and Eqs. (5)–(7). The T Lagrangians associated with either the constraints of Eq. (3) or Eq. (7) have some interesting properties which are included in Section 6. In the jargon of DEA, they are the 'virtual weights' or endogenously imputed levels of importance assigned to various time horizons for the j_0th fund. These weights can be useful for sensitivity purposes and for trading off performances over different horizons, with no change in a fund's score.

4. Identification of slacks and further discrimination; a lexiographic, pre-emptive programming approach

In Approach I for example, it is of course possible that a given fund could have a θ^* equal to 1 or greater and still have further slacks possible. This can happen, for example, in the radial augmentation formulation of Eqs. (1)–(4) if, for at least one of the time horizons, the inequality in Eq. (2) or Eq. (3) is not tight or binding. For example, if the left side of Eq. (3) is strictly more than the right side of Eq. (3) for some t, then even though the radial factor θ^* is maximized, there can still be a so-called 'slack' or possible improvement in the j_0th fund's mean returns for at least one of its horizons. The same type of situation can occur between the left and right sides of Eq. (2) where, if the left side of Eq. (2) is strictly less than $\sigma^2_{j_{0,t}}$ for some $t = \tau$ then some strict reduction in risk is possible for the τth horizon, with no degradation in any of the mean returns and no increases in any of the other risks.

To re-emphasize the appeal of Approaches I and II, they did not require any prior tradeoffs concerning the relative importance of the different horizons. However, to help identify say the truly maximum increases possible in mean returns for a given fund, *over and above the radial augmentation level*, we shall apply a standard device used in DEA (see, for example, Ref. [2]), known as lexicographic, pre-emptive programming which takes advantage of the fact that there may be a multiplicity of optimal solutions to a given formulation. Use of this tool does require a simple *ordering* of the relative meaningfulness of the various horizons, but not explicit tradeoffs.

Suppose the importance orderings for the T horizons are $\tau_1, \tau_2, \ldots, \tau_T$. Then the basic method of attack (say for Approach I) requires for each fund the solving of $2T+1$ different math programming formulations where the solutions from the previous formulations become constraints in the subsequent formulations.

To begin with, for the j_0 fund, we first solve Formulation I, i.e. Eqs. (1)–(4), yielding a θ^*.

The next step is to solve a second quadratic program, namely with constraints (1)–(3), as before, but where now $\theta = \theta^*$. The new objective is now the maximization of the mean return for only the most important facet, that is:

$$\text{Max} \sum_{j=1}^{N} w_j E(R_{j,\tau_1}) \tag{9}$$

Suppose the optimal amount of Eq. (9) is denoted $M(\tau_1)$. Then the next math program to be solved has as constraints those before, together with Eqs. (1)–(3), where $\theta = \theta^*$ and

$$\sum_{j=1}^{N} w_j E(R_{j,\tau_1}) = M(\tau_1) \tag{10}$$

with the new objective function being:

$$\text{Max} \sum_{j=1}^{N} w_j E(R_{j,\tau_2}) \tag{11}$$

namely, the mean return for the second most important horizon. Then, solving these additional T programs yields all the slacks possible in the mean returns.

Then, if one so desires, one can next focus on any slacks possible in the variances (i.e. risks). For example, one could next solve *another* T quadratic programs where one locks in the above means returns and θ^* and then identifies any risk reduction potentials possible over the various horizons.

When one concludes these calculations, involving solving a total of $2T+1$ programs, only those funds with $\theta^* = 1$, and *with no slacks whatsoever*, would constitute the truly *undominated* funds.

Hence, we can now summarize how a complete ranking of the N mutual funds can be accomplished, using *either* radial efficiency scores (i.e. either θ^* or Z^*), as well as any accompanying 'slacks'. For concreteness, suppose we desire the rankings from the perspective of the mean return augmentation (i.e. Formulation I). (An analogous approach works for the risk contraction approach). Then the steps are: (1) first rank the N funds on the magnitude of their θ^*'s alone, with the higher values of θ^* being associated with poorer performances; (2) if all the funds have different θ^* scores, one is done; (3) suppose, on the other hand, there are 'ties' in terms of the θ^* score (this will occur most often when $\theta^* = 1$). Then we shall use the magnitudes of any *additional percent increases in the mean return for the most important horizon* (over and above that arising from the *simultaneous* augmentation) to break the tie.

The fund with the *smallest* percent increase possible is then deemed the best of the tied funds, since the 'distance' to its maximum targets is less. If a tie still persists, then the same procedure is repeated for the second most important horizon, etc. This type of tie breaking logic can be repeated for all T horizons, focusing on the mean returns and if necessary on the risk reduction potentials over of the various horizons. These calculations will be illustrated in Section 5 where it is shown all 26 funds' performances can be completely discriminated.

Section 5 illustrates the notions of Sections 3 and 4, using 26 actual funds.

5. Numerical illustrations

5.1. Data

For the empirical results we selected 26 mutual funds to evaluate using our various approaches. In order to be considered in our numerical illustrations, each mutual fund had to meet *both* of the following two criteria:

(i) The funds had to have at least 10 years of monthly return data available. This was done so that we could examine each mutual fund's 10-year returns. Because our sample period ended on June 30, 1995, each fund selected had an inception date at or before July 1, 1985.

(ii) The fund had to be classified as of June 30, 1995 by Morningstar as '*aggressive growth*'. An aggressive growth fund according to Morningstar's On-Disk manual [19], "seeks rapid growth of capital, often through investment in smaller companies and with investment techniques involving greater than average risk, such as short-selling, leveraging and frequent trading". Further it should be mentioned that Morningstar classified mutual funds into their particular investment class based either on the wording in the prospectus issued by the mutual fund's advisor, or to a lesser extent, on the manner in which the fund was marketed.

Our reasoning for using aggressive growth funds for our empirical work was that there were only a limited number of 'aggressive growth' funds with 10 years or more of monthly returns. As a result we could examine an entire class of funds without an onerous amount of effort[4]. Hence the results of our exercises provide insights only on these aggressive funds. In order to gain insight on the relative attractiveness of other types of funds, vis-á-vis aggressive funds, one could use a stratified sample of funds, with representatives from different classes. Then for those classes of funds looking particularly attractive, the analysis could be repeated to identify the best individual fund performers within the most attractive classes.

For each of the 26 mutual funds, we calculated, for *each* of the 3, 5 and 10-year time periods, using monthly percentage return data:

(i) Mean monthly returns.
(ii) Covariances.
(iii) Variances.

These values were calculated using monthly return data from Morningstar's Mutual Funds On-Disk program. Morningstar calculates these returns by taking the change in monthly net asset value, reinvesting all income and capital-gains distributions during that month, and dividing by the starting net asset value. Note that these returns *do account* for management, administrative, 12b-1 fees and other costs automatically taken out of fund assets. The returns are not adjusted for sales charges[5]. The twenty-six funds analyzed are presented in Table 1 together with their 3-year, 5-year and 10-year mean monthly returns.

5.2. Examples of insights for one fund, State Street Research Capital C Fund

For each of the 26 funds, we initially ran both Approach I and Approach II. To better understand the results from the two approaches, consider the complete analysis for one particular fund, namely State Street Research Capital C (our 21st fund). Table 2 summarizes the actual performances of the State Street Fund.

Morningstar, in their proprietary rating system, gave this fund '5 stars', their highest rating. Furthermore, their detailed scoring system (to be discussed subsequently) would rank it the 4th best among the 26 funds considered[6]. The State Street Fund had a Morningstar final score of 1.262 whereas the median score for the 26 funds was about.185. Finally consider

[4] For example, an examination of funds classified as 'growth' funds with 10 years of monthly returns would have meant a sample of well over 100 funds.

[5] See Morningstar's Mutual Fund On-Disk Manual (Ref. [19], pp. 119–120).

[6] Briefly, what Morningstar does is generate a proprietary 'Morningstar return' and 'Morningstar Risk' for each mutual fund over three time horizons: 3-year, 5-year and 10-year. Then they subtract each time horizons' Morningstar risk from the Morningstar return. The resulting values are then weighted by giving the 10-year number a 50% weighting, the 5-year number a 30% weighting and the 3-year number a 20% weighting. The resulting number is then plotted along a bell curve to determine the fund's ranking: If the fund scores in the top 10% of it's class (aggressive growth), it receives 5 stars; if it falls in the next 22.5%, it receives 4 stars; a place in the middle 35% earns it 3 stars; if it falls in next 22.5%, it receives 2 stars; and the bottom 10% receive only 1 star (see Ref. [19] for more information).

Table 1
Funds analyzed and their mean monthly returns (all horizons end on June 30, 1995)

Fund No. and name	3 Year mean monthly return	5 Year mean monthly return	10 Year mean monthly return
(1) 20th Century Ultra Investors	1.636	1.813	1.737
(2) 44 Wall Street Equity	1.419	1.065	1.074
(3) AIM Aggressive Growth	2.786	2.175	1.543
(4) AIM Constellation	1.822	1.625	1.791
(5) Alliance Quasar A	1.022	0.593	1.033
(6) Delaware Trend A	1.550	1.398	1.463
(7) Evergreen Aggressive Grth A	1.416	1.518	1.368
(8) Founders Special	1.261	1.388	1.367
(9) Fund Manager Aggressive Grth	0.969	0.908	0.985
(10) IDS Strategy Aggressive B	0.847	0.815	1.165
(11) Invesco Dynamics	1.600	1.486	1.303
(12) Keystone Amer Omega A	1.075	1.166	1.349
(13) Keystone Small Co Grth (S-4)	1.966	1.690	1.411
(14) Oppenheimer Target A	1.166	1.101	1.114
(15) Pacific Horizon Aggr Growth	1.050	1.125	1.385
(16) PIMCo Adv Opportunity C	2.136	2.018	1.659
(17) Putnam Voyager A	1.458	1.411	1.510
(18) Security Ultra A	1.013	0.611	0.735
(19) Seligman Capital A	0.852	1.053	1.145
(20) Smith Barney Aggr Growth A	1.513	1.118	1.479
(21) State St. Research Capital C	1.966	1.663	1.619
(22) SteinRoe Capital Opport	1.591	1.131	1.169
(23) USAA Aggressive Growth	1.286	1.061	0.979
(24) Value Line Leveraged Gr Inv	1.338	1.090	1.215
(25) Value Line Spec Situations	1.305	0.893	0.850
(26) Winthrop Focus Aggr Growth	1.294	1.325	1.145

The 3 covariance matrices (3, 5 and 10 year horizons) are in Appendix B. The diagonals of these matrices, of course, yield the respective variances.

the Sharpe indices for the State Street Fund: for the three time horizons, the State Street Fund would rank *third* for the 3-year horizon; *11th* for the 5-year horizon and *fourth* for the 10-year horizon, out of the 26 funds. This variation in performance highlights the difficulty in arriving at a single summary score for a given fund.

Consider first the insights from exercising Approach I for the State Street Fund; the value of θ^* for the State Street Fund is 1.0556 (see Table 5), meaning that there is a convex combination of the other 25 funds, which could increase the mean monthly returns for *each* of the 3 time horizons by *at least* 5.56%, without increasing any of the actual variances actually experienced by the State Street Fund. Indeed Approach I finds that the composite portfolio of mutual funds (its endogenous benchmark), from the Approach I evaluation of the State Street Fund, is constructed by using the following convex weighting of the other 25 funds: $w_3^* = 0.1986$ (AIM Aggressive), $w_4^* = 0.5738$ (AIM Constellation), $w_{16}^* = 0.2166$ (PIMCO Adv Opportunity C) and $w_{17}^* = 0.0113$ (Putnam Voyage A), with all other w's at the zero level. This composite fund's frontier returns and risks are listed in Table 3.

Observe from Table 3 that the custom tailored benchmark fund (for the evaluation of the State Street Fund) had higher mean returns, and no higher variances than did the State Street Fund. Secondly, we observe the benchmark fund had a 5.56% increase in its mean monthly return for the 3-year and 10-year horizons, whereas the percent increase for the 5-year horizon was over 9.6%, i.e. well in excess of the 5.56%. This excess of some 4% represents the ‘slack’ discussed earlier, and is the amount over and above the *maximum simultaneous* increase possible. Note the variances for the frontier composite fund were the same as those for the State Street Fund for the 3-year and 10-year horizons and even less (18.3% less) for the 5-year horizon. Hence the reduction of 6.92 units of risk in State Street’s 5 year returns is another type of ‘slack’ that would need to be eliminated if this fund were completely undominated. This score of 1.0556, if

Table 2
Actual performances of State Street Fund (all horizons end on June 30, 1995)

	3-Year horizon	5-Year horizon	10-Year horizon
Mean monthly return (%)	1.966	1.663	1.619
Variance of monthly returns	17.86	37.96	41.48
Range of covariances (for other 25 funds)	7.75–19.88	14.6–36.9	25.44–44.55
Sharpe ratios	0.39	0.21	0.18

used as a ranking device, would rank the State Street Fund as only *ninth* out of 26 (compared to Morningstar's #4 rank). It is important to stress that such rankings need to be viewed cautiously since the composite benchmark funds used to arrive at each fund's q are typically quite different. Clearly though, the State Street Fund is unmistakably dominated by the composite of the four actual funds, a 'fund of funds' that was actually realizable.

Next consider a similar exercise for Approach II; for State Street, the risk contraction score (Z^*) is 0.8803 (see Table 5), ranking it (coincidentally) also ninth out of 26. Hence the interpretation here is that there is a composite of the 25 other funds for which *all* three variances could be reduced (relative to State Street's) by at least 1–0.8803 or about 12%, without lowering *any* of the mean returns. Note again the 'slack' in the 5 year horizon variance of State Street (see Table 4) where the percent reduction for the 5 year horizon is actually about 28%, well in excess of the 12%, realizable for the 3 and 10 year horizons. The maximum additional slacks possible will be addressed in Section 5.4. The benchmark frontier fund also has changed somewhat (from Approach I); it now consists of the 5 funds (instead of 4 earlier), the weightings being:

$$w_3^* = 0.191, \quad w_4^* = 0.2982, \quad w_{16}^* = 0.2083,$$

$$w_{17}^* = 0.2595 \quad \text{and} \quad w_{26}^* = 0.0349$$

5.3. Ratings and frontier values for the 18 clearly dominated funds

We first exercised Approach I and Approach II on each of the 26 funds in our sample. For eight of the funds, no convex combination of the other 25 funds could yield either an augmentation factor larger than 1, or a risk contraction factor less than 1. These 8 funds are then deemed superior to the other 18 funds (to be further discriminated subsequently). We also mention that the proportion of funds with $\theta^* = 1$, or $Z^* = 1$ would most likely shrink considerably as we increase the number of funds that are candidates for our endogenous benchmark creation. Table 5 summarizes, for each of the 18 dominated funds, its actual returns and variances, the ratings or scores for each under the two approaches, and the frontier levels for each horizon. The asterisks in Table 5 denote whether the corresponding constraint in either Approach I or II was binding or not, at optimality. For example, for the State Street Fund, the asterisks are on the 3-year and 10-year horizon levels as those are the horizons for which the constraints in Eqs. (2) and (3) or in Eqs. (6) and (7) were binding.

5.4. Identification of maximum slacks possible and further discrimination among the 8 'tied' funds

The next steps were: (1) to identify the maximum slacks possible for the clearly dominated funds and (2) to discriminate among those 8 funds where no simultaneous augmentation of the mean returns was possible, nor any simultaneous contraction was otherwise possible (see Section 5.5).

Table 3
Comparison of frontier and actual levels of risks and returns for State Street Fund (from Approach 1)

	3-Years	5-Years	10-Years
Actual monthly mean returns for State Street Fund	1.966	1.663	1.619
Frontier (benchmark fund) mean returns of Composite Fund	2.08	1.82	1.71
Actual monthly variances of returns for State Street Fund	17.86	37.96	41.48
Frontier (benchmark fund) variances of returns for Composite Fund	17.86	31.04	41.48

Table 4
Comparison of frontier and actual levels of risks and returns for State Street Fund (under risk contraction focus, approach II)

	3-Years	5-Years	10-Years
Actual monthly mean returns for State Street Fund	1.966	1.663	1.619
Frontier mean variances	1.966	1.750	1.619
Actual monthly variance of Returns for State Street Fund	17.86	37.96	41.48
Frontier variance of returns	15.72	27.50	36.51

The strategy used to accomplish both of the purposes was that presented in Section 4, namely the lexiographic, pre-emptive approach where the subjective ordering on the relative importances of the different horizons was taken (for concreteness) to be 10 years, 5 years and 3 years (following that of Morningstar).

To illustrate the additional maximum slacks possible for a given typical fund, we return to the State Street Fund (the 21st fund), where we have seen the $\theta^* = 1.0556$. Using the return augmentation formulation, the question now being addressed is "Are there a *different* set of weights that give rise to exactly the same θ^*, but which also maximizes any *excess* mean return possible (i.e. the 'slack') for the 10 year horizon?" (We observe this excess mean return is currently at zero since the inequality in Eq. (3) is tight for the 10 year horizon). When this program was solved, the result was that indeed there was no further increase possible for the 10 year mean return. The next step is then to focus on the 5 year mean return, holding θ^* at its current level. When this was done, the mean return for the 5 year horizon was found to be able to be further increased to 1.867, a 2.6% increase over that identified in Eqs. (1)–(4) (i.e. the earlier value of 1.82 resulted for the 5 year mean return with no emphasis on any slacks). Finally, to complete the analysis for State Street, a further slight increase in the 3 year mean returns was also possible, (namely to 1.717, from 1.710, holding the 5 year mean return now at 1.867). No reductions in any of the variances over the various horizons were further possible, holding the 10, 5 and 3 year mean returns at these new levels. The new composite benchmarking mutual fund has as weights: $w_3^* = 0.21$ (originally at 0.1986); $w_4^* = 0.160$ (originally at 0.5738); $w_{16}^* = 0.16$ (originally at 0.2166) and $w_{26}^* = 0.0113$ (originally $w_{17}^* = 0.0113$). This completes the slack analyses potential for The State Street Fund and illustrates the types of additional analyses possible, over and above that from focusing solely on the maximum simultaneous increase possible.

5.5. Breaking the 'ties' among the eight funds with no simultaneous augmentation or contraction possible

Finally, consider the further discrimination between the 8 funds with $\theta^* = 1$, and $Z^* = 1$, namely funds 1, 2, 3, 4, 9, 16, 17 and 26. To accomplish this, we first ran the formulation of Eqs. (1)–(3), $\theta^* = 1$, with the objective being to maximize $\Sigma_{j=1}^{N} w_j E(R_{j,10})$, i.e. the focus being on the 10 year horizon as it was assumed (following Morningstar) that this horizon was the most important of the 3.

When this was done, the percent increases possible in just the 10 year return *with no degradation in either* of the 5 or 3 years mean returns, and with and no increases in *any* of the variances, were: 1% for fund 16, 1.8% for fund 17, 3.1% for fund 1, 8.2% for fund 6, 15.3% for fund 3 and 66.7% for fund 2.

No increases were possible for funds 4 and 9 in the 10 year mean return. Hence if one was interested in possibly 'breaking the ties' for the 8 funds, then with the appropriate caveats, one obtains that funds 4 and 9 are the two 'best', followed by funds 16, 17, 1, 26, 3 and 2 in that order. This of course is all subject to the input that the 10 year horizon is the most important of the 3 horizons in terms of mean return improvement, and that the additional percent improvement in the 10 year mean return is a suitable criterion for breaking the tie. As mentioned earlier, one must exercise caution in doing so as the composite benchmarking funds can be quite different.

Finally, to discriminate among funds 4 and 9, we turned our focus to the 5 year return (locking in their 10 year mean returns) and found that a 28.1% further improvement was possible for fund #4 and a further 76.1% for fund #9. Hence, of these two, fund #4 might reasonably be deemed the better. Note also the very interesting point that *none* of the 26 are strictly undominated, as there is a composite fund for each that does strictly better in at least one of the six facets (i.e. the 3 mean returns and the 3 variances) and no worse in any of the other facets.

Table 5
Returns, frontier values and scores for the 18 clearly dominated mutual funds sample period: July 1, 1985 TO June, 30 1995 frequency: monthly

Fund name	Type of return	Mean monthly returns			Variances of monthly returns			Scores
		3 year	5 year	10 year	3 year	5 year	10 year	
Alliance Quasar A	actuals	1.02	0.59	1.03	14.54	25.88	38.51	NA
	return augmentation frontier	1.68	1.54	1.66*	13.50	25.58*	38.51*	1.602
	risk contraction frontier	1.39	1.13	1.07	5.85	13.69*	20.61*	0.535
Delaware Trend A	actuals	1.55	1.40	1.46	15.38	33.43	45.71	NA
	return augmentation frontier	1.86	1.65*	1.72*	15.38*	28.99	42.88	1.178
	risk contraction frontier	1.55*	1.48	1.46*	10.17*	19.77	30.22*	0.661
Evergreen	actuals	1.42	1.52	1.37	23.20	31.28	42.65	NA
	return augmentation frontier	2.12	1.87*	1.69*	18.90	31.28*	39.86	1.234
	risk contraction frontier	1.57	1.52*	1.37*	10.78	19.29*	26.29*	0.616
Founders Special	actuals	1.26	1.39	1.37	14.02	22.76	31.74	NA
	return augmentation frontier	1.64	1.56*	1.54*	12.87	22.76*	31.74*	1.126
	risk contraction frontier	1.45	1.41	1.37*	9.90	18.68*	26.06*	0.821
IDS Strategy B	actuals	0.85	0.82	1.17	13.35	22.08	29.87	NA
	return augmentation frontier	1.58	1.52	1.51*	12.83	22.08*	29.87*	1.292
	risk contraction frontier	1.21	1.17	1.17*	7.48	15.99*	21.63*	0.724
Invesco Dynamics	actuals	1.60	1.49	1.30	13.11	24.27	37.64	NA
	return augmentation frontier	1.96	1.74*	1.53*	13.11*	24.27*	34.79	1.172
	risk contraction frontier	1.60*	1.52	1.30*	8.13	16.46*	27.09*	0.739
Keystone America A	actuals	1.08	1.17	1.35	11.62	22.43	31.01	NA
	return augmentation frontier	1.53	1.46	1.51*	11.62*	21.50	31.01*	1.121
	risk contraction frontier	1.41	1.36	1.35*	9.59*	18.45	25.60*	0.825
Keystone Small Co.	actuals	1.97	1.69	1.41	23.21	39.32	43.37	NA
	return augmentation frontier	2.30*	1.98*	1.65*	19.25	32.04	39.79	1.169
	risk contraction frontier	1.97*	1.73	1.41*	13.99	24.58	30.06*	0.693
Oppenheimer Target	actuals	1.17	1.10	1.11	9.32	18.29	29.63	NA
	return augmentation frontier	1.49*	1.43	1.42*	9.19	18.29*	29.63*	1.278
	risk contraction frontier	1.20	1.20	1.11*	5.69	13.47*	21.82*	0.736
Pacific Horizon	actuals	1.05	1.13	1.39	18.81	35.42	40.74	NA
	return augmentation frontier	1.93	1.76	1.74*	18.81*	31.01	40.74*	1.257
	risk contraction frontier	1.47	1.43	1.39*	10.55	19.44*	26.43*	0.865
Security Ultra A	actuals	1.01	0.61	0.74	12.58	39.80	43.03	NA
	return augmentation frontier	2.10*	1.78	1.52*	12.58*	25.23	36.86	2.070
	risk contraction frontier	1.05	1.08	1.01	5.94	14.05	20.32*	0.472
Seligman Capital A	actuals	0.85	1.05	1.15	12.71	22.87	30.90	NA
	return augmentation frontier	1.57	1.50	1.53*	12.71*	22.41	30.90	1.332
	risk contraction frontier	1.18	1.15	1.15*	7.30	15.76*	21.29*	0.689
Smith Barney	actuals	1.51	1.12	1.48	22.13	33.46	45.08	NA
	return augmentation frontier	1.83	1.64	1.79*	16.87	30.87	45.08*	1.208
	risk contraction frontier	1.57	1.48	1.48*	9.63*	19.86	32.87	0.794
State Street	actuals	1.97	1.66	1.62	17.86	37.96	41.48	NA
	return augmentation frontier	2.08*	1.82	1.71*	17.86*	31.04	41.48*	1.056
	risk contraction frontier	1.97*	1.75	1.62*	15.72*	27.50	36.51*	0.880
Stein Roe Capital	actuals	1.59	1.13	1.17	14.62	28.34	38.18	NA
	return augmentation frontier	2.15*	1.82	1.58*	14.62*	27.74*	18.18*	1.352
	risk contraction frontier	1.59*	1.38	1.19	8.14	18.20*	24.52*	0.642
USA A	actuals	1.29	1.06	0.98	20.01	35.77	38.34	NA
	return augmentation frontier	2.18*	1.94	1.66	20.01*	31.71	38.34*	1.697
	risk contraction frontier	1.29*	1.16	1.09	7.63	16.70	21.31*	0.556
Value Line Leveraged	actuals	1.34	1.09	1.22	11.07	19.43	27.33	NA
	return augmentation frontier	1.55*	1.51	1.41*	10.83	19.43*	27.33*	1.161
	risk contraction frontier	1.34*	1.30	1.22*	7.92	16.23*	22.82*	0.835
Value Line Special	actuals	1.31	0.89	0.85	18.73	26.23	36.26	NA
	return augmentation frontier	2.28*	1.91	1.48*	14.17	26.23*	26.26*	1.746
	risk contraction frontier	1.31*	1.20	1.11	6.87	15.63*	21.61*	0.596

(An asterisk indicates the corresponding constraint in approach I or II is binding.)

Table 6
Rankings of funds from different approaches

Fund No. and Name	Ranking from return augmentation perspective	Ranking from risk contraction perspective	Morningstar rank (and # of stars)
(1) 20th Century	5	6	5 (5 stars)
(2) 44 Wall St.	8	8	18 (3 stars)
(3) AIM Agg	7	7	1 (5 stars)
(4) AIM Constill	1	1	3 (5 stars)
(5) Alliance Quasar	23	25	25 (2 stars)
(6) Delaware Trend	15	20	11 (3 stars)
(7) Evergreen	17	22	13 (3 stars)
(8) Founders	11	13	10 (4 stars)
(9) Fund Manager	2	2	17 (3 stars)
(10) IDS	20	17	22 (2 stars)
(11) Invesco	14	15	8 (4 stars)
(12) Keystone America	10	12	14 (3 stars)
(13) Keystone Small	13	18	7 (4 stars)
(14) Oppenheimer	15	16	19 (3 stars)
(15) Pacific Horizon	18	10	20 (3 stars)
(16) PIMCO	3	3	2 (5 stars)
(17) Putnam Voy	4	4	6 (5 stars)
(18) Security Ultra	26	26	26 (1 star)
(19) Seligman	21	19	21 (2 stars)
(20) Smith Barney	16	14	16 (3 stars)
(21) State Street	9	9	4(5 stars)
(22) SteinRoe	22	21	15 (3 stars)
(23) USAA	24	24	23 (2 stars)
(24) Value Line Leveraged	12	11	12 (3 stars)
(25) Value Line Special	25	23	24 (2 stars)
(26) Winthrop	6	5	9 (4 stars)

The same philosophy was applied for Approach II to break the tie among the 8 funds for which no simultaneous contraction was possible in all of the risks. The results were: a 2% risk contraction for the 10 year horizon was possible for fund #9; a 3% risk contraction for the 10 year horizon was possible for fund #16; a 10% risk contraction for the 10 year horizon was possible for fund #26; a 26% risk contraction for the 10 year horizon was possible for fund #1; a 31% risk contraction for the 10 year horizon was possible for fund #3; a 67% risk contraction for the 10 year horizon was possible for fund #2.

For funds #4 and 9, no risk contraction was possible for the 10 year horizon (with no decrease in any of the mean returns and no increase in either of the risks for the 5 and 3 year horizons). To further discriminate among these 2 funds, we next focused on the 5 year contraction potential; fund 9 had a 42% contraction possible, with fund 2 having a 44% contraction possible. These tie breaking results are incorporated in Table 6, which compares, for each of the 26 funds, the rankings from 3 different approaches: (i) Approach I; (ii) Approach II and (iii) Morningstar's overall ranking.

Upon inspection of Table 6, we observe that Approach I and Approach II yield different rankings[7]. For example, Delaware Trend is 15th based on return augmentation, but 20th from a risk contraction perspective; moreover, note Delaware Trend A is the 11th ranked fund from the Morningstar perspective. Recall our State Street Fund, which ranked 4th on Morningstar criterion, was only 9th from Approaches I and II. One consistency is that Security Ultra Fund, fund #18, is ranked last on all *three* criteria. Finally note the slightly different rankings (for the 8 tied funds) from the two different approaches.

[7] The two approaches yield somewhat different rankings but the Kendall Rank correlation coefficient between the two rankings is 0.81; hence the two orderings are broadly quite similar.

Table 7
Lagrangians for State Street Funds, return augmentation focus (formulation I)

	3 Years	5 Years	10 Years
Average monthly return (Eq. (3))	0.1045	0	0.4907
Variance of monthly returns (Eq. (2))	0.0073	0	0.0072

6. Lagrangian analyses

Both of the two basic approaches, that is Eqs. (1)–(4) and Eqs. (5)–(8), yield as byproducts, Lagrangians related to the constraints of Eqs. (1)–(3) and Eqs. (5)–(7). The T Lagrangians associated with the constraints in Eq. (3) have an interesting property. Referring to them as λ_t^* ($t = 1, 2, \ldots, T$), they satisfy;

$$\sum_{t=1}^{T} \lambda_t^* E(R_{j_{0,t}}) = 1 \tag{12}$$

(For a demonstration of this, see Appendix A).

These λ_t^*, in the jargon of data envelopment analysis, are the 'virtual weights' or endogenously imputed levels of importance assigned to the mean rates of return actually achieved for the tth horizon for the j_0th fund. Hence the formulation of Eqs. (1)–(4), rather than weighting the various mean return performances for the Fund over different time horizons using apriori weights, assigns its own weights endogenously. The weights assigned are chosen to give the j_0th fund manager the 'benefit of doubt' in the sense that any other set of weights would give rise to the same or an even *poorer rating* for the j_0th fund manager. In this spirit then, the resulting scores should be viewed as conservative.

In a similar fashion, using the formulation of Eqs. (5)–(8), the T Lagrangian values associated with the T variance constraints of Eq. (7), calling these u_t^*, have the appealing property that:

$$\sum_{t=1}^{T} u_t^* \sigma_{j_{0,t}} = 1 \tag{13}$$

These weights can be useful to exploring substitution possibilities or in gaining a feel as to how these horizon weightings compare, for example, to those of Morningstar's. To illustrate these ideas, Table 7 presents the Lagrangians related to the formulation of Eqs. (1)–(4) for our State Street case.

To illustrate first their sensitivity uses, consider the 0.0073 for the 3 year variance. If The State Street Fund's actual monthly variance for the 3 year horizon had been one unit *higher* at 18.86 (in contrast to its actual of 17.86), then the optimal value of θ could have been increased an additional 0.0073 units, i.e. from $\theta = 1.0556$ to $\theta = 1.0629$, i.e. the rating would have *worsened* by an additional 0.0073 units. The zeros signify, of course, which constraints in Eqs. (2) and (3) are non-binding or slack. Similarly, to illustrate the 0.1045 Lagrangian on the 3 year mean return, if the fund's actual 3 year mean return say were *decreased* 0.1 unit to 1.8667 (from 1.9667), then the rating (or scoring of the fund) would be $1.0556 + 0.01045 = 1.0661$.

As indicated by Eq. (12), the Lagrangian values for the Average monthly returns, when multiplied by the actual mean monthly returns, sum to 1, i.e. 0.1045 (1.9667) + 0 (1.6633) + 0.4907 (1.6192) = 1. Note that the so-called 'virtual' weight (by those practitioners of data envelopment analysis) for the State Street Fund's 5 year mean return is zero, consistent with the earlier finding that an additional slack (over and above the simultaneous augmentation amount) is present; this is a facet where the State Street's manager was particularly weak, on a relative basis.

These weights are also useful for exploring substitution possibilities, with no *change* in a fund's relative performance. For example, taking the ratio of the Lagrangians for the 3 year mean return and the 10 year mean return (respectively of 0.1045 and 0.4907, or 4.695) together with (12), yields the insight that one could exchange a 0.1 *increase* in the State Street Fund's 10 year return (from 1.6192 to 1.7192) with a 0.4695 *decrease* in the fund's 3 year mean return (i.e. from 1.9667 to 1.4977), all with *no* change in the fund's overall rating of 1.0556. Similar types of insights are available related to substitution of risks.

7. Caveats and conclusions

We have presented some new approaches for the identification of dominated mutual funds which assess and integrate the relative performances of a given fund over different time horizons. All of a fund's actual mean returns and risks over the different time horizons are combined into a single summary score with a clear economic interpretation. For the basic Approaches I and II, no a priori weights are needed regarding the relative importances of performances over different time horizons. Further, the procedure arrives at the levels of both mean returns and risks, for each time horizon, needed for the dominated fund *not* to have been so rated. Importantly, the endogenously created benchmarking fund used to arrive at the above insights was actually achievable in that one could have actually purchased this 'fund of funds', i.e. the frontier is realizable. This is in contrast to many DEA applications (with hospitals, schools, hotels, banks, etc.), where it is moot if the composite, benchmarking unit is indeed realizable.

Moreover, those ratings for each fund manager are conservative in that no other set of priorities (regarding the horizon importances) would yield any better assessment of the individual fund manager. The corollary of this is those fund managers found inefficient from Formulation I or II, in our case exactly two-thirds of them, cannot argue that the choice of weights on the various horizon performances were arbitrary and were biased against them. Secondarily, we can discriminate among those funds with no simultaneous improvement possible, but it does entail an additional subjective ordering of the importances.

While the above properties of our approaches are desirable, they, as with others, are not a panacea and have a number of caveats:

(i) Ratings based on past performances may not be a useful prediction of the future. In this regard, Lipper recently completed an in-house study where they found that Morningstar's 5-star funds actually performed quite poorly after the time they were classified as 5-star funds. The usefulness of our ratings needs to be explored also.

(ii) Our measure of risk is total risk, not just systematic risk. On the other hand, our endogenous benchmarking fund of funds compares the total risks and mean performances of it with those of the fund being evaluated over exactly the same time horizons. Additionally, the possible lowering of every total risk for a given fund (over every horizon), with no lowering of *any* of the mean returns, cannot be explained as simply due to more diversification (i.e. being compared to a 'fund of funds'), as sometimes the endogenous benchmark is just one other fund.

(iii) The approach (as with others) requires much data (e.g. all the covariances for *all* the time horizons being analyzed) and use of time consuming, quadratic programming optimization codes. In our case with 26 funds and 3 time horizons, both quadratic formulations required solving optimizations with a linear objective function, and with seven constraints, of which 3 were quadratic. Each of the quadratic constraints had 351 terms in it, i.e. the 26 w_j^2's, plus 325 of the pairs, w_i times w_j, multiplied by the covariances. Using EXCEL SOLVER, these were solved to optimization in an average of about 3 min per run. However, if one were ranking, say, hundreds of funds simultaneously, the number of variables and computer time would grow exponentially. Hence it may be best, as we did, to do the rankings within a given class of funds, or to include representative funds from the different fund types.

These and other caveats notwithstanding, we feel the new approaches, together with all of the side information provided, represent another novel application of the basic philosophy of data envelopment analysis and may provide another set of useful insights into the important task of rating/rating mutual funds. This is a part of the economy with literally trillions of dollars, showing no signs of slowing down. Our approaches incorporate the critically important time dimensions, an aspect that hitherto has defied aggregation, without resorting to the use of subjective weights. More work on this important and difficult problem is clearly needed.

Acknowledgements

This research was partially funded by TIAA-CREF's Research Section, directed by Mark Warshawsky, and the authors gratefully acknowledge his support and encouragement. Professor Gary Thompson at Cornell University was helpful in instructing the authors on Excel Solver and its use in quadratic optimizations.

Appendix A. Lagrangian analysis of Formulation 1

The formulation of Eqs. (1)–(4) can be solved by maximizing, with respect to θ, λ_t ($t = 0, \ldots, T$) and α_t ($t = 1, 2, \ldots, T$), the following:

$$\Phi = \theta + \lambda_0\left(\sum_{j=1}^{N} w_j - 1\right) + \sum_{t=1}^{T}\lambda_t\left(\sum_{j=1}^{N} w_j E(R_{j,t}) - \theta E(R_{j_{0,t}})\right) - \sum_{t=1}^{T}\alpha_t\left(\sum_{j=1}^{N} w_j^2\sigma_{j,t}^2 + \sum_{\substack{i=1 \\ i\neq j}}^{N}\sum_{j=1}^{N} w_i w_j \operatorname{Cov}(R_{i,t}, R_{j,t}) - \sigma_{j_{0,t}}^2\right)$$

Upon setting $\partial\Phi/\partial\theta = 0$, one obtains Eq. (12).

In a similar fashion, using the formulation of Eqs. (5)–(8), the T Lagrangian values associated with the T variance constraints of Eq. (7), calling them u_t^*, satisfy Eq. (13).

Appendix B

The covariances for 3-year time horizon (sample period: July 1992 to June 1995), 5-year time horizon (sample period: July 1990 to June 1995) and 10-year time horizon (sample period: July 1985 to June 1995) are shown in Table 8 (overleaf).

Table 8

Fund No.	Fund No. 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Covariances for 3-year time horizon. Sample period: July 1992 to June 1995																										
1	21.82	9.41	17.63	18.02	15.62	15.51	19.45	15.00	9.00	15.21	15.66	14.06	20.80	11.42	18.07	21.66	14.34	13.76	14.42	17.83	18.13	15.57	17.80	13.84	17.57	8.10
2	9.41	9.54	8.96	9.02	9.11	9.58	10.37	8.53	5.26	8.03	8.05	7.72	9.95	6.46	9.61	11.03	8.65	8.01	8.07	9.79	9.76	8.43	10.03	7.43	8.77	4.92
3	17.63	8.96	17.75	16.25	14.48	14.19	17.51	12.79	7.57	13.64	13.82	12.39	19.08	9.78	15.85	19.70	13.21	13.71	12.71	15.82	16.27	14.57	17.31	11.64	16.45	7.40
4	18.02	9.02	16.25	16.71	14.05	13.88	17.82	14.13	8.62	14.14	14.03	12.07	18.42	11.10	15.79	19.13	13.22	13.17	13.46	16.23	16.27	14.13	17.03	12.13	16.04	7.75
5	15.62	9.11	14.48	14.05	14.54	13.69	15.72	12.45	8.07	12.66	12.66	11.29	16.49	9.40	15.41	16.37	12.63	12.17	11.61	16.05	14.96	13.41	15.19	10.72	14.28	7.09
6	15.51	9.58	14.19	13.88	13.69	15.38	15.54	12.28	7.60	12.45	12.74	11.17	16.08	9.42	15.87	16.49	12.23	12.37	11.89	15.79	14.82	12.95	14.74	10.51	14.10	6.43
7	19.45	10.37	17.51	17.82	15.72	15.54	23.20	15.18	9.11	15.80	15.82	13.25	20.28	11.70	17.00	20.97	15.15	14.03	15.39	16.91	17.73	15.84	18.10	12.68	16.83	8.46
8	15.00	8.53	12.79	14.13	12.45	12.28	15.18	14.02	8.56	12.66	12.33	10.19	15.38	10.24	14.11	15.88	11.91	11.38	12.43	14.89	14.04	12.37	14.74	10.60	13.75	6.87
9	9.00	5.26	7.57	8.62	8.07	7.60	9.11	8.56	8.54	7.70	7.34	7.02	9.45	6.40	8.39	10.18	7.17	6.89	7.12	8.97	9.10	7.58	8.03	7.05	7.77	4.77
10	15.21	8.03	13.64	14.14	12.66	12.45	15.80	12.66	7.70	13.35	12.16	9.97	15.66	9.68	14.07	15.67	12.11	11.21	11.87	14.81	14.13	12.85	15.04	10.45	13.77	6.67
11	15.66	8.05	13.82	14.03	12.66	12.74	15.82	12.33	7.34	12.16	13.11	10.68	15.94	9.51	14.48	16.22	11.81	11.34	11.61	14.30	14.07	12.20	14.19	10.39	13.54	6.86
12	14.06	7.72	12.39	12.07	11.29	11.17	13.25	10.19	7.02	9.97	10.68	11.62	14.64	7.69	12.38	15.65	10.60	9.76	10.09	11.79	13.32	10.88	12.41	10.00	11.92	5.95
13	20.80	9.95	19.08	18.42	16.49	16.08	20.28	15.38	9.45	15.66	15.94	14.64	23.21	11.37	18.56	22.52	15.09	14.59	15.04	18.87	19.13	16.33	18.91	13.86	18.93	8.75
14	11.42	6.46	9.78	11.10	9.40	9.42	11.70	10.24	6.40	9.68	9.51	7.69	11.37	9.32	10.89	11.77	9.36	8.95	9.42	11.96	10.84	8.86	11.15	8.69	10.15	5.12
15	18.07	9.61	15.85	15.79	15.41	15.87	17.00	14.11	8.39	14.07	14.48	12.38	18.56	10.89	18.81	18.21	13.98	13.46	13.22	18.11	16.63	14.70	16.47	12.36	15.95	7.56
16	21.66	11.03	19.70	19.13	16.37	16.49	20.97	15.88	10.18	15.67	16.22	15.65	22.52	11.77	18.21	25.66	15.62	15.64	15.55	17.70	19.88	17.00	19.54	15.22	19.73	8.76
17	14.34	8.65	13.21	13.22	12.63	12.23	15.15	11.91	7.17	12.11	11.81	10.60	15.09	9.36	13.98	15.62	12.80	11.11	11.17	14.40	13.83	12.42	14.31	10.28	13.62	6.57
18	13.76	8.01	13.71	13.17	12.17	12.37	14.03	11.38	6.89	11.21	11.34	9.76	14.59	8.95	13.46	15.64	11.11	12.58	10.87	14.47	13.28	11.82	14.27	9.59	13.54	6.47
19	14.42	8.07	12.71	13.46	11.61	11.89	15.39	12.43	7.12	11.87	11.61	10.09	15.04	9.42	13.22	15.55	11.17	10.87	12.71	13.55	13.32	11.89	14.13	10.30	13.07	6.26
20	17.83	9.79	15.82	16.23	16.05	15.79	16.91	14.89	8.97	14.81	14.30	11.79	18.87	11.96	18.11	17.70	14.40	14.47	13.55	22.13	17.01	15.01	17.46	12.20	17.33	7.91
21	18.13	9.76	16.27	16.27	14.96	14.82	17.73	14.04	9.10	14.13	14.07	13.32	19.13	10.84	16.63	19.88	13.83	13.28	13.32	17.01	17.86	14.30	16.66	12.77	16.12	7.75
22	15.57	8.43	14.57	14.13	13.41	12.95	15.84	12.37	7.58	12.85	12.20	10.88	16.33	8.86	14.70	17.00	12.42	11.82	11.89	15.01	14.30	14.62	15.78	10.66	15.02	6.83
23	17.80	10.03	17.31	17.03	15.19	14.74	18.10	14.74	8.03	15.04	14.19	12.41	18.91	11.15	16.47	19.54	14.31	14.27	14.13	17.46	16.66	15.78	20.01	12.37	17.62	7.59
24	13.84	7.43	11.64	12.13	10.72	10.51	12.68	10.60	7.05	10.45	10.39	10.00	13.86	8.69	12.36	15.22	10.28	9.59	10.30	12.20	12.77	10.66	12.37	11.07	11.68	6.13
25	17.57	8.77	16.45	16.04	14.28	14.10	16.83	13.75	7.77	13.77	13.54	11.92	18.93	10.15	15.95	19.73	13.62	13.54	13.07	17.33	16.12	15.02	17.62	11.68	18.73	6.83
26	8.10	4.92	7.40	7.75	7.09	6.43	8.46	6.87	4.77	6.67	6.86	5.95	8.75	5.12	7.56	8.76	6.57	6.47	6.26	7.91	7.75	6.83	7.59	6.13	6.83	5.56
Covariances for 5-year time horizon. Sample period: July 1990 to June 1995																										
1	39.47	13.38	33.06	32.80	27.17	30.93	31.21	26.72	20.84	27.36	28.70	27.21	37.29	23.65	35.12	33.86	26.98	32.90	26.53	31.77	36.21	28.51	33.99	25.38	28.57	17.68
2	13.38	10.71	14.20	13.46	13.40	14.86	12.89	12.04	9.15	11.52	12.10	11.17	14.84	10.42	13.67	13.65	11.86	15.39	11.84	13.96	14.60	13.69	14.42	10.55	11.80	8.15
3	33.06	14.20	34.50	30.93	27.86	31.10	28.28	23.81	18.68	25.08	27.05	24.47	35.12	22.14	31.72	31.58	25.39	33.34	24.78	30.56	33.47	28.03	32.23	22.57	27.85	17.30
4	32.80	13.46	30.93	30.90	25.78	29.59	28.81	24.50	19.69	25.14	25.97	24.04	33.77	22.39	30.78	30.37	25.04	31.59	25.14	29.36	32.57	27.61	31.53	22.76	26.09	17.21
5	27.17	13.40	27.86	25.78	25.58	27.54	23.90	21.41	17.01	21.71	23.09	21.40	29.25	19.10	27.41	26.41	21.82	27.97	21.31	26.94	28.46	24.61	26.68	19.34	23.45	15.80
6	30.93	14.86	31.10	29.59	27.54	33.43	27.44	24.15	18.72	24.50	26.34	23.46	32.96	21.51	31.71	29.70	24.53	33.79	24.41	30.17	32.20	28.02	30.33	21.42	25.82	17.46
7	31.21	12.89	28.28	28.81	23.90	27.44	31.28	23.33	18.52	24.37	24.63	22.60	31.90	20.75	28.93	28.58	24.17	28.07	24.42	26.96	30.68	26.35	29.79	21.39	24.69	16.32
8	26.72	12.04	23.81	24.50	21.41	24.15	23.33	22.76	16.87	20.83	21.53	19.89	27.04	18.47	25.42	24.93	20.43	25.38	21.01	24.11	26.22	22.70	25.14	18.98	21.29	14.08
9	20.84	9.15	18.68	19.69	17.01	18.72	18.52	16.87	16.93	16.80	16.96	16.90	21.41	15.46	19.42	19.25	16.73	19.38	16.59	18.99	21.94	18.10	20.14	16.14	16.22	12.64
10	27.36	11.52	25.08	25.14	21.71	24.50	24.37	20.83	16.80	22.08	21.78	20.02	27.94	18.70	26.04	24.50	21.25	25.58	21.13	24.93	27.19	23.21	26.61	19.13	21.80	14.56

11	28.70	12.10	27.05	25.97	23.09	26.34	24.63	21.53	16.96	21.78	24.27	21.20	29.23	19.14	27.15	26.68	21.53	27.96	21.11	25.79	28.85	23.23	26.93	19.67	22.85	15.20
12	27.21	11.17	24.47	24.04	21.40	23.46	22.60	19.89	16.90	20.02	21.20	22.43	27.54	17.64	25.15	25.90	20.31	23.70	20.14	23.40	27.17	21.49	24.43	19.27	20.99	14.51
13	37.29	14.84	35.12	33.77	29.25	32.96	31.90	27.04	21.41	27.94	29.23	27.54	39.32	24.11	35.10	34.60	27.78	34.84	27.64	33.24	36.90	30.55	34.74	25.40	29.98	19.00
14	23.65	10.42	22.14	22.39	19.10	21.51	20.75	18.47	15.46	18.70	19.14	17.64	24.11	18.29	22.57	21.04	19.10	23.09	19.03	22.22	23.91	19.87	23.09	17.32	18.91	12.90
15	35.12	13.67	31.72	30.78	27.41	31.71	28.93	25.42	19.42	26.04	27.15	25.15	35.10	22.57	35.42	30.86	26.08	32.34	25.60	31.69	34.04	27.88	32.04	23.47	27.36	17.57
16	33.86	13.65	31.58	30.37	26.41	29.70	28.58	24.93	19.25	24.50	26.68	25.90	34.60	21.04	30.86	34.97	24.65	30.73	24.44	29.23	33.78	26.69	30.54	23.35	27.35	17.04
17	26.98	11.86	25.39	25.04	21.82	24.53	24.17	20.43	16.73	21.25	21.53	20.31	27.78	19.10	26.08	24.65	22.47	25.57	21.25	24.99	27.22	23.10	26.48	19.20	21.90	14.53
18	32.90	15.39	33.34	31.59	27.97	33.79	28.07	25.38	19.38	25.58	27.96	23.70	34.84	23.09	32.34	30.73	25.57	39.80	25.57	31.11	33.96	29.85	33.16	22.41	26.83	17.45
19	26.53	11.84	24.78	25.14	21.31	24.41	24.42	21.01	16.59	21.13	21.11	20.14	27.64	19.03	25.60	24.44	21.25	25.57	22.87	24.10	26.66	23.13	25.94	18.99	21.16	14.37
20	31.77	13.96	30.56	29.36	26.94	30.17	26.96	24.11	18.99	24.93	25.79	23.40	33.24	22.22	31.69	29.23	24.99	31.11	24.10	33.46	32.10	26.69	30.94	22.09	26.61	17.24
21	36.21	14.60	33.47	32.57	28.46	32.20	30.68	26.22	21.94	27.19	28.85	27.17	36.90	23.91	34.04	33.78	27.22	33.96	26.66	32.10	37.96	29.19	33.94	25.17	28.38	18.95
22	28.51	13.69	28.03	27.61	24.61	28.02	26.35	22.70	18.10	23.21	23.23	21.49	30.55	19.87	27.88	26.69	23.10	29.85	23.13	26.69	29.19	28.34	29.15	20.54	23.95	15.89
23	33.99	14.42	32.23	31.53	26.68	30.33	29.79	25.14	20.14	26.61	26.93	24.43	34.74	23.09	32.04	30.54	26.48	33.16	25.94	30.94	33.94	29.15	35.77	23.90	27.46	17.61
24	25.38	10.55	22.57	22.76	19.34	21.42	21.39	18.98	16.14	19.13	19.67	19.27	25.40	17.32	23.47	23.35	19.20	22.41	18.99	22.09	25.17	20.54	23.90	19.43	19.62	13.64
25	28.57	11.80	27.85	26.09	23.45	25.82	24.69	21.29	16.22	21.80	22.85	20.99	29.98	18.91	27.36	27.35	21.90	26.83	21.16	26.61	28.38	23.95	27.46	19.62	26.23	14.40
26	17.68	8.15	17.30	17.21	15.80	17.46	16.32	14.08	12.64	14.56	15.20	14.51	19.00	12.90	17.57	17.04	14.53	17.45	14.37	17.24	18.95	15.89	17.61	13.64	14.40	12.89
Covariances for 10-year time horizon. Sample period: July 1985 to June 1995																										
1	54.33	45.82	45.20	47.19	41.19	45.19	42.06	37.58	27.88	36.13	42.37	37.79	46.37	34.26	43.40	40.77	37.94	41.59	37.22	45.80	44.45	41.58	42.07	35.37	41.06	30.94
2	45.82	61.66	40.77	42.69	39.41	42.86	37.51	33.48	25.45	32.69	39.39	34.30	39.51	32.70	35.46	34.06	34.47	36.49	33.08	42.01	38.93	37.76	36.42	32.14	37.98	31.58
3	45.20	40.77	44.25	42.32	38.25	41.51	37.72	32.73	24.73	32.14	38.33	32.41	41.70	31.22	36.70	37.30	34.06	39.42	33.29	41.08	39.53	37.51	38.41	30.57	37.34	28.50
4	47.19	42.69	42.32	45.58	39.22	42.63	40.38	35.33	26.74	34.76	39.59	34.92	42.82	32.69	39.49	37.89	35.87	40.23	35.97	42.51	41.01	39.71	39.72	33.42	38.22	29.91
5	41.19	39.41	38.25	39.22	38.51	39.90	35.65	31.93	24.07	30.90	35.10	30.97	37.88	30.64	34.77	33.81	32.13	36.54	31.49	38.80	36.84	35.38	34.95	29.06	34.62	28.47
6	45.19	42.86	41.51	42.63	39.90	45.71	38.40	34.73	25.81	33.48	38.63	33.25	41.33	32.70	38.34	36.94	34.86	40.31	34.53	42.12	40.08	38.82	38.51	31.46	37.39	30.01
7	42.06	37.51	37.72	40.38	35.65	38.40	42.65	31.99	23.87	32.89	34.95	30.87	38.71	30.53	35.54	34.38	32.63	35.53	32.68	37.68	37.21	35.86	36.25	29.51	34.23	27.29
8	37.58	33.48	32.73	35.33	31.93	34.73	31.99	31.74	22.20	28.18	31.44	28.09	34.19	27.24	31.44	30.57	29.21	32.00	29.45	33.93	32.54	31.72	31.59	27.29	30.70	24.65
9	27.88	25.45	24.73	26.74	24.07	25.81	23.87	22.20	18.99	21.16	23.88	21.94	25.77	21.34	22.74	22.78	21.99	24.38	21.89	25.45	25.44	23.66	23.79	21.11	22.86	19.14
10	36.13	32.69	32.14	34.76	30.90	33.48	32.89	28.18	21.16	29.87	30.50	27.34	33.60	26.62	31.44	29.40	28.49	30.81	28.37	33.34	32.36	30.94	31.66	26.33	29.83	23.67
11	42.37	39.39	38.33	39.59	35.10	38.63	34.95	31.44	23.88	30.50	37.64	31.26	38.14	29.31	34.60	34.09	31.90	36.53	31.35	38.30	36.52	34.82	35.32	29.74	34.67	26.79
12	37.79	34.30	32.41	34.92	30.97	33.25	30.87	28.09	21.94	27.34	31.26	31.01	34.06	26.27	30.95	29.92	28.27	30.72	28.04	33.13	32.96	29.99	30.58	27.32	29.97	23.52
13	46.37	39.51	41.70	42.82	37.88	41.33	38.71	34.19	25.77	33.60	38.14	34.06	43.37	31.92	38.89	37.54	34.56	39.23	34.43	41.07	40.81	37.89	38.60	31.93	37.46	28.66
14	34.26	32.70	31.22	32.69	30.64	32.70	30.53	27.24	21.34	26.62	29.31	26.27	31.92	29.63	28.27	27.84	27.05	31.15	27.13	31.49	31.21	28.39	29.79	25.29	28.89	24.60
15	43.40	35.46	36.70	39.49	34.77	38.34	35.54	31.44	22.74	31.44	34.60	30.95	38.89	28.27	40.74	33.47	31.95	34.97	31.76	38.65	37.77	35.09	35.29	29.64	33.58	25.34
16	40.77	34.06	37.30	37.89	33.81	36.94	34.38	30.57	22.78	29.40	34.09	29.92	37.54	27.84	33.47	36.33	30.61	34.51	30.21	35.97	36.00	33.56	34.00	28.42	33.74	25.67
17	37.94	34.47	34.06	35.87	32.13	34.86	32.63	29.21	21.99	28.49	31.90	28.27	34.56	27.05	31.95	30.61	30.55	32.01	29.45	34.99	33.21	32.27	32.44	27.31	31.37	24.54
18	41.59	36.49	39.42	40.23	36.54	40.31	35.53	32.00	24.38	30.81	36.53	30.72	39.23	31.15	34.97	34.51	32.01	43.03	32.42	38.35	37.37	35.97	36.37	29.22	34.57	27.06
19	37.22	33.08	33.29	35.97	31.49	34.53	32.68	29.45	21.89	28.37	31.35	28.04	34.43	27.13	31.76	30.21	29.45	32.42	30.90	34.05	32.69	32.02	31.78	27.19	30.61	24.38
20	45.80	42.01	41.08	42.51	38.80	42.12	37.68	33.93	25.45	33.34	38.30	33.13	41.07	31.49	38.65	35.97	34.99	38.35	34.05	45.08	39.49	38.05	38.38	31.64	37.38	28.68
21	44.45	38.93	39.53	41.01	36.84	40.08	37.21	32.54	25.44	32.36	36.52	32.96	40.81	31.21	37.77	36.00	33.21	37.37	32.69	39.49	41.48	36.27	37.11	30.91	35.46	28.02
22	41.58	37.76	37.51	39.71	35.38	38.82	35.86	31.72	23.66	30.94	34.82	29.99	37.89	28.39	35.09	33.56	32.27	35.97	32.02	38.05	36.27	38.18	35.71	29.18	34.12	26.46
23	42.07	36.42	38.41	39.72	34.95	38.51	36.25	31.59	23.79	31.66	35.32	30.58	38.60	29.79	35.29	34.00	32.44	36.37	31.78	38.38	37.11	35.71	38.34	29.57	34.32	26.37
24	35.37	32.14	30.57	33.42	29.06	31.46	29.51	27.29	21.11	26.33	29.74	27.32	31.93	25.29	29.64	28.42	27.31	29.22	27.19	31.64	30.91	29.18	29.57	27.33	28.68	22.80
25	41.06	37.98	37.34	38.22	34.62	37.39	34.23	30.70	22.86	29.83	34.67	29.97	37.46	28.89	33.58	33.74	31.37	34.57	30.61	37.38	35.46	34.12	34.32	28.68	36.26	26.34
26	30.94	31.58	28.50	29.91	28.47	30.01	27.29	24.65	19.14	23.67	26.79	23.52	28.66	24.60	25.34	25.67	24.54	27.06	24.38	28.68	28.02	26.46	26.37	22.80	26.34	24.69

References

[1] Banker R, Kaufman R, Morey RC. Measuring gains in operational efficiency from information technology. A study of the position deployment at Hardee's. J Management Information Syst 1990;7(2):29–54.

[2] Banker RD, Morey RC. Efficiency analysis for exogenously fixed inputs and outputs. Oper Res 1986;34(4):513–21.

[3] Charnes A, Cooper WW, Rhodes E. Evaluating program and managerial efficiency: an application of data envelopment analysis to program follow through. Management Sci 1981;27:668–97.

[4] Connor G, Korzjszyk R. Performance measurement with the arbitrage pricing theory: a new framework for analysis. Rev Quant Finance Account 1991;March 7:4–25.

[5] Elton E, Gruber M, Blake C. The persistence of risk-adjusted mutual fund performance. J Business 1996;69:133–60.

[6] Elton E, Gruber M, Das S, Hlavka M. Efficiency with costly information: a reinterpretation of evidence from manager portfolios. Rev Finan Stud 1993;6:1–22.

[7] Farrell MJ. The measurement of productive efficiency. J R Stat Soc Ser A 1957;120:253–90.

[8] Grinblatt M, Titman S. Mutual fund performance: an analysis of quarterly portfolio holdings. J Business 1989;62:393–416.

[9] Grinblatt M, Titman S. Performance measurement without benchmarks: an examination of mutual fund returns. J Business 1993;66(1):47–68.

[10] Jensen M. The performance of mutual funds for the period 1945–1964. J Finance 1968;23(2):389–416.

[11] Jensen M. Risk, the pricing of capital assets, and the evaluation of investment portfolios. J Finance 1969;April:167–247.

[12] Joy M, Porter R. Stochastic dominance and mutual fund performance. J Finan Quant Anal 1974;January 9(1):25–31.

[13] Lehman B, Modest D. Mutual fund performance evaluation: a comparison of benchmarks and benchmark comparisons. J Finance 1987;June:233–65.

[14] Lewin A, Morey RC, Cook T. Evaluating the administrative efficiency of courts. Omega 1982;Spring 82:401–11.

[15] Lintner J. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. Rev Econ Stat 1965;47:13–37.

[16] Markowitz H. Portfolio selection. J Finance 1952;March:77–91.

[17] Meyer J. Further application of stochastic dominance to mutual fund performance. J Finan Quant Anal 1977;June:235–42.

[18] Morey RC, Ozcan Y, Retzlaff-Roberts D, Fine D. Estimating the hospital-wide differentials warranted for teaching hospitals: an alternative to regression approaches. Med Care 1995;5(33):531–52.

[19] Morningstar's Mutual Fund On-Disc Manual. Morningstar, Chicago, IL, 1995.

[20] Murthi BPS, Choi YK, Desai P. Efficiency of mutual funds and portfolio measurement: a non-parametric approach. Eur J Oper Res 1997;98:408–18.

[21] Okunev J. An alternative measure of mutual fund performance. J Business Finance Account 1990;Spring 17(2):247–64.

[22] Roll R. Ambiguity when performance is measured by the security market line. J Finance 1978;September 33:1051–69.

[23] Ross S. The arbitrage theory of capital asset pricing. J Econ Theory 1976;December:341–60.

[24] Sharpe WF. Mutual fund performance. J Business 1966;January:119–38.

[25] Sharpe W. Determining a fund's effective asset mix. Invest Manage Rev 1988;June:5–15.

[26] Time Magazine. Sept. 30, 1996.

[27] Treynor J. How to rate management investment funds. Harvard Business Rev 1965;January–February:63–75.

[28] Jaffe C. Rating the raters: flaws found in each service. Boston Globe, August 27, 1995, p. 78.

[29] Gould C. Star ratings don't tell entire story. New York Times, July 16, 1994, section 3, p. 5.