Bayesian Analysis of Systemic Risk Distributions

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Abstract

I propose Bayesian Markov Chain Monte Carlo (MCMC) estimation of systemic risks proposed in Brownlees and Engle (2012). The systemic risks are measured by MES (marginal expected shortfall), LRMES (Long Run Marginal Expected shortfall) and SRISK (expected capital shortage of a firm conditional on a substantial market decline). The analysis is performed incorporating Dynamic Conditional Correlation (DCC) model with asymmetric volatility using generalized threshold conditional volatility model (GTARCH). The analysis is compared with GJR-GARCH volatility model. The proposed model captures leverage effect (asymmetry) in both ARCH and GARCH terms. We find that distributions of out-of-sample volatility forecasts and MES risks are statistically different for highly ranked financial institutions in periods of low volatility using both DCC-GJR-GARCH and DCC-GTARCH models. However, LRMES distributions and SRISK distributions could be highly overlapping. Moreover, when volatility is high it is hard to rank financial institutions based on either volatility, MES, LRMES or SRISK measure as distributions overlap. The SRIKS measures become very close when leverage ratios of companies are similar. Thus, in order to distinguish systemic risk measures incorporating uncertainty additional factors, such as liquidity, are needed.

KEY WORDS: Markov Chain Monte Carlo; Systemic Risks prediction; Dynamic Conditional Correlation; Asymmetric GARCH; Metropolis-Hastings steps.

1 Introduction

After the 2008-2009 financial crisis the topic of systemic risk became increasingly more important in both academic research and public policy discussion. Regulators have been implementing new capital requirements, stress tests and "living wills" (resolution plans) for financial institutions. The Dodd Frank Act and Basel regulation are aimed at finding ways to make financial system more stable and resilient to major shocks, in particular, due to concentration of risk in large and interconnected financial institutions. While every financial crisis has its own major risk driver the common feature of crises is instability of some part of the financial system that serves as an important intermediary between real sector economy and investors. While a firm is affected by a crisis it depends on the interconnectedness of the firm with rest of the system how its potential bankruptcy may affect the rest of the economy. If a financial firm is large and highly interconnected failure of such firm causes considerable strain to the rest of the financial sector and a negative externality on the rest of the economy.[1] If a large interconnected firm experiences capital shortage it may not be able to raise capital on its own and may implicitly rely on government bailout using taxpayer funds.

Following recent studies of systemic risks by Acharya et. al (2010) and Brownlees and Engle (2012) among others I introduce Bayesian estimation of MES (marginal expected shortfall) and SRISK (expected capital shortage of a firm conditional on a substantial market decline). The rankings for MES and SRISK are used to analyze systemic risks of financial institutions and are daily reported by Volatility Institute[2]. However, this measures are reported without uncertainty around estimates and thus one cannot distinguish if the difference in rankings of large financial institutions is statistically significant. Many other measures were introduced in literature such as CoVaR (Adrian and Brunnrmeier (2011), systemic risk index CAITFIN (Allen et. al (2012), probability of default measures (Huang et. al 2011). These studies looked at contribution of a firm in distress to overal risk of the financial system. Recent surveys of systemic risk analytics by Bisias et al. (2012) and Brunnermeier and Oehmke (2012) among others also do not show how to measure and incorporate uncertainty for systemic risk measures. To fill this gap present paper shows how to estimate MES and SRISK using Bayesian Markov Chain Monte Carlo (MCMC) algorithms.

In this paper I also introduce a generalized threshold conditional volatility model (GTARCH) and compare it to traditional asymmetric models of volatility. Since introduction of the generalized autoregressive conditional heteroscedasticity (GARCH) model there have been many extensions of GARCH models that resulted in better statistical fit and forecasts. For example, GJR-GARCH (Glosten, Jagannathan, & Runkle (1993)) is one of well-known extensions of GARCH models with an asymmetric term which captures the effect of negative shocks in equity prices on volatility commonly referred to as a "leverage" effect. The widely used GJR-GARCH model has a problem that ARCH (α) coefficient tends to takes a meaningless negative value in unconstrained estimation of equity returns volatility. The typical solution to this problem is setting coefficient of alpha to zero in the constrained estimation.

In the proposed GTARCH model both coefficients, ARCH (α) and GARCH (β), are allowed to change to reflect the asymmetry of volatility due to negative shocks. As a subset of this model GJR-GARCH model allows for asymmetry only in ARCH. Alternatively, the GTARCH model allows for asymmetry only in GARCH or no asymmetry. Additional asymmetric GARCH term shifts the value of α upward compared to the GJR-GARCH model. In particular, unconstrained estimation may result in statistically significant negative α for

[1]Theoretical academic research showed this, for example, in the paper by Acharya, V., Pedersen, L., Philippon, T., and Richardson, M. (2010).

[2]See http://vlab.stern.nyu.edu

the GJR-GARCH model, while in the GTARCH model α is typically insignificant. The suggested more flexible GTARCH model also shows more persistent dynamics for GARCH parameters for negative news and lower persistence for positive news. Our results for equity returns show that compared to GJR-GARCH and GARCH our model predicts higher level of volatility in high volatility periods and lower levels of volatility in low volatility periods.

The GTARCH is used within DCC (dynamic conditional correlations model) in order to measure MES and SRISK. In order to estimate MES Brownlees and Engle (2012) first use the Maximum Likelihood estimation of GJR-GARCH volatility models for market and firm returns and then dynamic conditional correlation (DCC) model for tail dependence. MES can be derived as a function of volatility, correlation and tail expectations of a firm and market return innovations. When measuring tail expectation Brownlees and Engle (2012) use nonparametric kernel estimation without incorporating uncertainty. In this paper using Bayesian MCMC estimation I obtain distributions for parameters of interest including tail risk measures.

In this paper MCMC algorithms are used for estimation of all volatility models and distributions of systemic risks are derived from MCMC draws. The advantage of Markov Chain Monte Carlo algorithms is their natural ability to generate posterior predictive densities for variables of interest, such as volatility, correlation, value at risk, expected shortfall, etc. I use Metropolis-Hastings steps with random walk draws. The algorithms for estimating more general ARMA-GTARCH models are based on extension of algorithms in Goldman and Tsurumi (2005).

As an additional meaure of systemic risks I use Credit Default Swaps (CDS). CDS spreads are widely used to access default risks of financial institutions and sovereign bonds. the relation between CDS spreads, bond yield spreads and credit rating announcements. Carr and Wu (2011) show the relation between CDS spreads and out-of-the-money American put options. The CDS premiums change dramatically over time and may exhibit nonstationary behaviour. It can be argued that systemic risks of financial institutions can be related to the level and volatility of CDS premiums.[3] In this paper I estimate GTARCH model for the log-differences of CDS spreads and find that asymmetric reaction resulting from higher spread is better explained by the GTARCH than the GJR-GARCH model.

Overall, this paper offers the following contributions. First, I propose Bayesian estimation of a GTARCH model and compare its performance with traditional asymmetric volatility models. Second, the new model is applied for forecasting volatility of equities and log changes of CDS premiums. Fourth, the equity volatility forecasts combined with correlation with the market are used for the measurement of systemic risks, MES and SRISK, in a fashion similar to Brownlees and Engle (2012) but incorporating better asymmetric volatility properties and uncertainty for risk measures.

The remainder of the paper is organized as follows. Section 2 presents the measurements of the systemic risks and section 3 presents the GTARCH model. Section 4 presents summary statistics of the data for Bank of America (BAC), JP Morgan Chase (JPM), Citi

[3] Work in this direction was recently done by Oh and Patton (2013)

group (CIT) and S&P 500. Section 5 presents the MCMC algorithms. Section 6 estimates models using MCMC and shows distribution of systemic risk measures in periods of high and low volatility. Section 7 concludes.

2 Measurement of Systemic Risk

Let r_t and $r_{m,t}$ be the daily log returns of a firm and the market correspondingly. Following Brownlees and Engle (2012) I consider the following model for the returns:

$$\begin{aligned} r_{mt} &= \sigma_{mt}\epsilon_{mt} \\ r_t &= \sigma_t\rho_t\epsilon_{mt} + \sigma_t\sqrt{1-\rho_t^2}\epsilon_t \end{aligned} \tag{1}$$

where $\epsilon_{mt}, \epsilon_t$ are independent and identically distributed variables with zero means and unit variances, σ_t and σ_{mt} are conditional standard deviations of the firm return and the market return correspondingly, and ρ_t is conditional correlation between the firm and the market. This model is also called the dynamic conditional beta model with $\beta_t = \rho_t \dfrac{\sigma_t}{\sigma_{mt}}$ and tail dependence on correlation of firm returns and the market

$$r_t = \beta_t r_{mt} + \sigma_t\sqrt{1-\rho_t^2}\epsilon_t \tag{2}$$

The conditional variances and correlation are modelled using the GJR-GARCH DCC model in Brownlees and Engle (2012). In the next section I introduce the generalized threshold GARCH volatility model and show that it outperforms GJR-GARCH for equities.

In this paper I only consider the market based measures of systemic risks. Other macroprudential and microprudential tests are beyond the scope of this paper but are described in Bisias, Flood, Lo and Valavanis (2012) and Acharya, Engle and Pierret (2013) among others.

The first considered systemic risk measure is the daily marginal expected shortfall (MES) which is the conditional expectation of a daily return of a financial institution given that the market return falls below threshold level C. In practice, in VLAB it is assumed that market falls by more than 2%, i.e. the threshold $C = -2\%$.

$$\begin{aligned} MES_{t-1} &= E_{t-1}(r_t | r_{mt} < C) \\ &= \sigma_t\rho_t E_{t-1}(\epsilon_{mt}|\epsilon_{mt} \leq C/\sigma_{mt}) + \sigma_t\sqrt{1-\rho_t^2}E_{t-1}(\epsilon_t|\epsilon_{mt} \leq C/\sigma_{mt}) \end{aligned} \tag{3}$$

The computation of the expected shortfall following Scaillet (2005) using nonparametric estimates given by:

$$E_{t-1}(e_{mt}|e_{mt} \leq \alpha) = \frac{\sum_{i=1}^{t-1} e_{mi}\Phi_h(\frac{\alpha - e_{mi}}{h})}{\sum_{i=1}^{t-1}\Phi_h(\frac{\alpha - e_{mi}}{h})} \tag{4}$$

where $\alpha = C/\sigma_{mt}$, $\Phi_h(t) = \int_{-\infty}^{t/h} \phi(u)du$, $\phi(u)$ is a standard normal probability distribution function used as kernel, and $h = T^{-1/5}$ is the bandwidth parameter.

The second measure is the long run marginal expected shortfall based on the expectation of the cumulative six month firm return conditioned on the event that the market falls by more than d% (which by default is 40%) in six months.

$$LRMES_t = 1 - exp(ln(1-d) * \beta) \tag{5}$$

Finally, the capital shortfall of the firm based on the potential capital loss in six months is defined as

$$SRISK_t = max\{0; kD_t - (1-k)(1 - LRMES_t)E_t\} \tag{6}$$

where D_t is the book value of Debt at time t, E_t is the market value of equity at time t and $k \approx 8\%$ is the prudential capital ratio of the US banks. It is assumed that the capital loss happens only due to the loss in the market capitalization $LRMES * E_t$

3 Generalized Threshold GARCH model

GJR-GARCH (Glosten, Jagannathan, & Runkle (1993)) is one of the well-known asymmetric volatility models which captures the effect of negative shocks in equity prices on volatility commonly referred to as a "leverage" effect. The model captures risk-aversion of investors with volatility increasing more as a result of a negative news compared to the positive news.[4]

Consider the GJR-GARCH volatility model for returns r_t with mean μ given in equation (7) below.

GJR-GARCH(1,1,1)

$$\begin{aligned} r_t &= \mu + \epsilon_t \\ \sigma_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I(r_{t-1} - \mu < 0) + \beta\sigma_{t-1}^2 \end{aligned} \tag{7}$$

where I is a (0,1) indicator function, σ_t is conditional volatility.

The Generalized Threshold GARCH (GTARCH) model that I introduce in equation (8) is an extension of the model above allowing GARCH term to change for a negative news ($\epsilon_{t-1} < 0$).

GTARCH(1,1,1,1)

$$\begin{aligned} r_t &= \mu + \epsilon_t \\ \sigma_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I(r_{t-1} - \mu < 0) + \beta\sigma_{t-1}^2 + \delta\sigma_{t-1}^2 I(r_{t-1} - \mu < 0) \end{aligned} \tag{8}$$

[4] EGARCH is an alternative model but it is in logs of variance rather than typical GARCH variance.

The Stationarity of GTARCH Model

The weak stationarity condition in the GARCH model for the existence of the long run unconditional variance σ^2 is given by condition:

$$\alpha + \beta < 1, \qquad \sigma^2 = \frac{\omega}{1 - \alpha - \beta}$$

Similarly for the GTARCH model we can define $\theta = E(I(r_t < \mu))$ which is percentage of observations with $r_t < \mu$. Then the weak stationarity condition and the unconditional variance are given by

$$\alpha + \beta + \gamma\theta + \delta\theta < 1, \qquad \sigma^2 = \frac{\omega}{1 - \alpha - \beta - \gamma\theta - \delta\theta}$$

4 Data

In this section I consider the equity returns daily data for BAC, JPM, CIT and S&P 500 index for the period 1/04/2001-12/31/2012 from CRSP database. I also consider the CDS spreads on the 5 year secured bonds of BAC and JPM for the period 9/06/2001-10/08/2013 from Bloomberg. All these data will be used for the analysis of systemic risks in Section 6.

The summary statistics of the data are given in Table 1. All the series have fat tails with the kurtosis over 10 and some skewness. Even though the CDS spreads typically have significant positive skewness the log-differences of CDS spreads for BAC and JPM do not show considerable skewness.

There may be some autocorrelation present in the model although AR(1) coefficients are not large.

Table 1 here

I consider the GTARCH model for the returns and log-differences of CDS spreads. Unlike for the equity returns the bad news in CDS market is when the spreads increase. Thus, I change the sign of the error in the dummy indicator function to $I(r_{t-1} - \mu > 0)$ for the CDS data.

5 Markov Chain Monte Carlo Algorithms

Markov Chain Monte Carlo (MCMC) algorithms allow to estimate posterior distributions of parameters by simulation and are especially useful when the dimension of parameters is high, since the problems of multiple maxima or of initial starting values are avoided.

A simple intuitive explanation of the Metropolis-Hastings algorithm is given in Chib and Greenberg (1995).

MCMC algorithms were developed by Chib and Greenberg (1994) for the ARMA model and by Nakatsuma (2000) and Goldman and Tsurumi (2005) for the ARMA-GARCH model. Chib and Greenberg (1994) (as well as Nakatsuma (2000)) use the constrained nonlinear maximization algorithm in the MA block. Alternatively one can use a Metropolis-Hastings algorithm with a random walk Markov Chain as was done e.g. in Goldman and Tsurumi (2005). The random walk draws speed up the computational time of the MCMC algorithms without losing much of the acceptance rate of the Metropolis-Hastings algorithm. In this paper I propose the algorithms for a GTARCH model which is an extension of the algorithms developed in Goldman and Tsurumi (2005).

Let the prior probability for the GTARCH volatility model be given by

$$\begin{aligned} \pi(\mu, \alpha, \gamma, \beta, \delta) \quad &\propto \quad \mathrm{N}(\mu_0, \Sigma_\mu)\, \mathrm{N}(\alpha_0, \Sigma_\alpha)\, \mathrm{N}(\gamma_0, \Sigma_\gamma) \\ &\times \quad \mathrm{N}(\beta_0, \Sigma_\beta)\, \mathrm{N}(\delta_0, \Sigma_\delta) \end{aligned} \tag{9}$$

where μ, α, γ, β and δ are the GTARCH parameters and have proper normal priors with large variances.

Consider the Dynamic Conditional Correlations (DCC) model with GTARCH volatility. The posterior pdf of DCC model is

$$p(\eta_1, \eta_2, \psi|data) \propto \pi(\eta_1, \eta_2, \psi) \times L(data|\eta_1, \eta_2, \psi) \tag{10}$$

$$\eta_i = \mu_i, \alpha_i, \gamma_i, \beta_i, \delta_i$$

$$\psi = \omega_{ij}, \alpha, \beta$$

Let n=2 (2 firms, or one firm and a market).

The DCC log likelihood is given by

$$logL = log(L_v(\eta_1, \eta_2) + log(L_c(\eta_1, \eta_2, \psi) \tag{11}$$

$$log(L_v) = -0.5 \sum (nlog(2\pi) + log(\sigma_{i,t}^2) + \frac{r_{i,t}^2}{\sigma_{i,t}^2}) \tag{12}$$

$$log(L_c) \quad = \quad -0.5 \sum \left(log(1 - \rho_{12,t}^2) + \frac{z_{1,t}^2 + z_{2,t}^2 - 2\rho_{12,t} z_{1,t}^2 z_{2,t}^2}{1 - \rho_{12,t}^2} \right) \tag{13}$$

$$\rho_{12,t} \quad = \quad \frac{q_{12,t}}{\sqrt{q_{11,t} q_{22,t}}} \tag{14}$$

$$q_{ij,t} \quad = \quad \omega_{ij}(1 - \alpha - \beta) + \alpha z_{i,t} z_{j,t} + \beta q_{ij,t-1} \tag{15}$$

$$\tag{16}$$

where $r_{i,t}$ and $r_{m,t}$ are daily log returns of firm i and the market correspondingly. The standardized returns: $z_{i,t} = \dfrac{r_{i,t}}{\sqrt{h_{it}}}$

Step 1: I estimate parameters in blocks for each asset GTARCH model using random walk draws.

Step 2: using fitted volatilities from step 1 find standardized returns z_{it} and estimate dynamic correlation between two assets. I estimate parameters in blocks using random walk draw: (i) ARCH parameters: α and ω_{12} as part of ARCH, (ii) GARCH parameters β, (iii) Constant terms $\omega_{ii} = 1 - \alpha - \beta$ for i=1,2.

Each step is a separate MCMC chain and careful tests of convergence are applied.[5]

6 Data Analysis of MES and SRISK

I consider Bank of America, Citigroup and JP Morgan Chase ranked in the top three highest systemically important financial firms on VLAB website as of December 31,2012-June 7, 2013 (Tables 5-6).

Table 2 here

For the systemic risk modeling as in Brownlees and Engle (2012) I use market data on stock prices, market capitalization and book value of debt for large financial institutions. The data are from CRSP for returns and market capitalization for the period 2001/01/02-2012/12/31. The book value of debt is from COMPUSTAT.

The summary statistics of returns are given in Table 1, the results of Bayesian estimation of GTARCH volatility models are given in Table 3 and the results for the DCC correlation are given in Table 4. I presented the posterior means of parameters and 95% highest posterior density intervals (HPDI).

Tables 3 and 4 here

The dynamic volatility estimated at posterior means of parameters is plot in Figure 1. The correlation of firms with the market estimated at posterior means of parameters is given in Figure 2.

The marginal expected shortfall (MES) is given in Figure 3, LRMES in figure 4 and SRISK in Figure 5. All the graphs use posterior means of parameters and equations (3)-(6) for computation of the measures of interest.

[5] I use the graphs of draws, fluctuation test (see Goldman and Tsurumi (2005)) and the acceptance rates to judge convergence. The results are available from author on request.

Finally I consider a 1 day out-of-sample prediction of MES, LRMES and SRISK and derive the posterior distribution for each of these quantities using posterior distributions of $\sigma_{T+1}, \sigma_{m,T+1}, \rho_{T+1}$ obtained from the MCMC draws.

The practical implementation is as follows.

Figures 6,7,8 show the posterior pdfs of MES, LRMES and SRISK correspondingly for the first two firms listed in Table 4: BAC and JPM. It turns out that their measures of risk are statistically different with 95% HPDI's not crossing. This confirms that the rankings used on the VLAB website are distinguishing firms in terms of severity of the systemic risks they impose on the system.

Figure 1 shows the returns data for BAC, JPM and SPX. The dynamic GTARCH volatility estimated at posterior means of parameters is plotted in Figure 2. While before the financial crisis JPM had higher level of volatility, during the crisis and after the crisis BAC volatility level exceeded JPM. Not surprisingly the SPX has lower equity volatility then both banks. The dynamic correlation of firms with the market also estimated at posterior means of parameters is given in Figure 3. For comparison I also present 100-day rolling correlations in Figure 4. Both graphs show changing patterns of correlation over time with less variability for the DCC-GTARCH model.

After the equity volatility models were estimated for each bank I found the distributions of 1% Value at Risk (VaR) and showed them in Figure 5 for a \$1 million portfolio using (a) Normal distribution for the error term and (b) historical simulation of residuals (bootstrap). These pdfs of VaR show clearly that the VaR are statistically different for different distributional assumptions of the error term. Since the historical simulation shows significantly higher VaR it is preferable to use it rather than Normal distribution.

Figure 6 shows the CDS spreads and log-differences of CDS spreads. The CDS spreads for BAC and JPM seem to move together to some extent. As with equity volatility the CDS spreads were higher for JPM before the financial crisis and lower for the most time starting from the financial crisis. The log-differences of CDS spreads exhibit volatility clustering similar to equity returns. Figure 7 shows the leverage of BAC and JPM and the dynamics is similar to the CDS spreads with BAC leverage highly exceeding JPM leverage starting from the financial crisis.

The systemic risk measures of the marginal expected shortfall (MES), LRMES and SRISK over time are presented in Figures 8-10. All the graphs use posterior means of parameters of the DCC-GTARCH model and equations (3)-(6) for computation of the measures of interest. Half of the sample is used for MES of the first observation in 2006. We can see that the MES results also show higher risks for BAC starting from the crisis when BAC leverage increased dramatically and lower MES before the crisis. However, graphs are close and more careful analysis of the distributions of MES at a particular point is needed. Graphs of LRMES and SRISK show similar patterns with peaks during the financial crisis and potential treasury bonds default with debt ceiling reached in August 2011. The SRISK average values presented in Figure 10 are similar to values reported by VLAB such as in Table 5. For example, at the end of the sample (2012/12/31) SRISK is about 104.2 \$

billion for BAC and 80.2 $ billion for JPM using MCMC for the GJR-GARCH model as in Brownless and Engle (2012). The VLAB values are 101 $ billion for BAC and 75.8 $ billion for JPM.[6]

Finally I consider the whole posterior distribution for $MES_T, LRMES_T$ and $SRISK_T$ derived from the posterior distributions of $\sigma_T, \sigma_{m,T}, \rho_T$ obtained from the MCMC draws. Figure 11 shows the distribution of MES and SRISK for JPM at the end of the sample (T=2012/12/31) which is in the period of low volatility, while Figure 12 shows these measures in the period of high volatility (T=2008/08/29). I present the results when the GTARCH, GJR-GARCH and GARCH models are used. The interesting implication of the GTARCH model is that the results for volatility, MES and SRISK are lower in a period of low volatility and higher in a period of high volatility compared to GJR-GARCH and GARCH. GARCH model is less responsive than other two models to the periods of high and low volatility as it has no asymmetric news effect that captures risk-aversion. It seems that the TGARCH model captures risk-aversion better than GJR-GARCH model that is a commonly used model in the literature.[7]

For the remainder of the graphs I use the GTARCH model. Figures 13-15 compare the BAC and JPM posterior pdfs of MES, LRMES and SRISK for the low volatility time (T=2012/12/31). It turns out that their measures of risk are statistically different with distributions not crossing. This means that in the periods of low volatility the rankings of BAC being above JPM are justified distinguishing firms in terms of severity of the systemic risks they impose on the system. Figures 16-17 show MES and SRISK for JPM and BAC at the time of high volatility (T=2008/08/29) and we see that the MES distributions are close to each other with 95% highest posterior density intervals intersecting. JPM had higher leverage on that day and it resulted in somewhat higher SRISK but the results for BAC and JPM are not statistically significant. The results not presented here to save space indicate that the same pattern happens at other dates in periods of high volatility.

7 Conclusion

In this paper I considered Bayesian estimation of systemic risks. Using a new asymmetric GARCH model and capturing uncertainty around the measures I found that MES, LRMES and SRISK are statistically different for major financial firms at the times of low volatility, however, MES measures in particular may be very close at the times of uncertainty such as the financial crisis.

The paper has several contributions. This is the first paper to introduce Bayesian analysis for the systemic risk measures and derive the full distribution of those measures compared to simple point estimates used in the literature. Second, a new asymmetric GTARCH model introduced in this paper generalizes popular asymmetric volatility GJR-

[6] The results may the difference in estimation period used and constraints imposed on the GJR-GARCH model by the VLAB.

[7] The other asymmetric GARCH model is EGARCH

GARCH model and improves its properties. Third, I provide the whole distribution of systemic risk measures and show how to distinguish risks of different institutions. I also estimate GTARCH volatility of log-difference in CDS spreads showing alternative measures of financial risks.

For the future work I would like to consider different distributional assumptions for the error term. It would be also interesting to compare the market based measures of systemic risks used in this paper to the results of macroprudential stress tests.

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Table 1: Summary statistics for daily equity returns

	BAC	CIT	JPM	SPX
mean	0.045	-0.009	0.047	0.011
std	3.406	3.673	2.841	1.342
Skew	0.904	1.468	0.829	0.017
Kurt	26.08	42.668	15.931	11.143
AR(1)	-0.011	0.046	-0.089	-0.091

Notes: Equity returns are measured in basis points. Equity prices data are for the period 1/04/2001-12/31/2012 from CRSP database.

Table 2: VLAB Systemic Risks for US institutions

Institution	SRISK%	RNK	SRISK ($ m)	LRMES	Beta	Cor	Vol	Lvg
			29-Aug-08					
Citigroup	12.89	1	132,039	73.64	2.61	0.79	63.4	19.99
JPMorgan Chase	9.37	2	96,045	70.95	2.42	0.74	62.9	13.42
Bank of America	9.24	3	94,637	77.27	2.9	0.74	75	11.94
Freddie Mac	6.74	4	69,069	92.26	5.01	0.44	221.2	297.76
American International Group	6.62	5	67,811	83.01	3.47	0.69	97	17.62
Merrill Lynch	6.6	6	67,588	82.66	3.43	0.78	83.8	22.45
Fannie Mae	6.56	7	67,156	94.01	5.51	0.51	205.4	115.68
Morgan Stanley	6.39	8	65,416	65.62	2.09	0.74	53.7	23.01
Goldman Sachs	5.63	9	57,676	58.04	1.7	0.75	43.3	16.99
Wachovia Bank	5.09	10	52,131	79.05	3.06	0.66	87.3	22.4
Lehman Brothers	4.71	11	48,249	92.18	4.99	0.74	130.2	55.88
MetLife	2.4	12	24,589	51.59	1.42	0.79	34.4	14.56
Prudential Financial	2.12	13	21,714	49.82	1.35	0.72	36.1	15.39
Washington Mutual	2.03	14	20,787	74.95	2.71	0.45	119.8	41.5
			31-Mar-09					
Bank of America	17.16	1	160,739	85.42	3.77	0.74	195.9	48.7
Citigroup Inc	14.12	2	132,262	85.42	3.77	0.66	219.3	121.21
JPMorgan Chase	13.91	3	130,281	75.58	2.76	0.8	133.1	20.11
Wells Fargo	8.94	4	83,752	80.99	3.25	0.73	170.5	20.53
American International Group	6.21	5	58,141	75.83	2.78	0.44	252.2	55.88
Goldman Sachs	5.47	6	51,257	60.93	1.84	0.8	88.3	18.58
Morgan Stanley	4.28	7	40,046	73.09	2.57	0.77	127.6	24.41
MetLife	3.64	8	34,045	80.2	3.17	0.73	168.5	26.12
Prudential Financial	3.45	9	32,280	88.65	4.26	0.74	221	52.34
Hartford Financial	2.25	10	21,095	84.34	3.63	0.72	194.2	106.08
			31-Dec-12					
Bank of America	17.98	1	100,700	53.52	1.5	0.66	28.8	16.4
Citigroup Inc	15.03	2	84,188	48.26	1.29	0.66	24.9	16.02
JPMorgan Chase	14.81	3	82,949	43.57	1.12	0.75	18.8	13.69
MetLife	8.62	4	48,306	56.95	1.65	0.73	28.6	22.75
Goldman Sachs	7.62	5	42,680	52.08	1.44	0.74	24.6	15.14
Prudential Financial	7.05	6	39,517	51.59	1.42	0.75	23.2	26.44
Morgan Stanley	6.93	7	38,838	51.62	1.42	0.69	25.9	19.42
Hartford Financial	3.34	8	18,721	54.23	1.53	0.73	26.3	30.17
American International Group	2.34	9	13,109	52.56	1.46	0.62	29.4	9.6
Lincoln National	2.31	10	12,925	52.81	1.47	0.75	24.7	29.11

Source: http://vlab.stern.nyu.edu

Table 3: Estimation results for various volatility models

	GTGARCH	GJR-GARCH	GTGARCH_0	GARCH
μ	0.035 (0.016)	0.030 (0.016)	0.040 (0.017)	0.043 (0.016)
ω	0.037 (0.005)	0.034 (0.004)	0.039 (0.005)	0.036 (0.005)
α	0.059 (0.011)	0.062 (0.010)	0.071 (0.010)	0.116 (0.011)
γ	0.055 (0.024)	0.103 (0.018)		
β	0.856 (0.012)	0.867 (0.010)	0.850 (0.011)	0.860 (0.011)
δ	0.076 (0.026)		0.111 (0.023)	
$\alpha + \beta + .5(\gamma + \delta)$	0.981 (0.006)	0.981 (0.005)	0.977 (0.006)	0.976 (0.005)
vol. forecast $\sqrt{252h_{T+1}}$ (%)	15.05 (0.62)	13.95 (0.27)	15.53 (0.718)	13.27 (0.216)
1% VaR ($)	23.28 (0.75)	22.24 (0.48)	24.10 (0.92)	21.76 (0.39)
Correl $(r_{t-1}, log(h_t/h_{t-1}))$	-0.513	-0.434	-0.426	-0.115
MBIC at mean	3411.55	3412.85	3416.44	3437.01
MBIC at mode	3369.41	3375.40	3380.10	3406.52

Notes: Data for the S&P500 index for the period 01/04/2001-12/31/2012. All coefficients are reported at posterior means and standard deviations are given in brackets. All parameters are statistically significant, i.e. the 95% Highest Posterior Density Intervals (not reported to save space) do not include zero. I derive posterior distributions of 1 day out of sample volatility forecast ($\sqrt{252h_{T+1}}$) and of Value at Risk (VaR) using MCMC draws of parameters. The 1% VaR is constructed for $1000 portfolio for 1 day out of sample forecast and is corrected for fat tails using historical simulations. MBIC is the Modified Bayesian Information Criterion.

Table 4: Estimation results for DCC-GJR-GARCH model

	BAC	CIT	JPM
ω_{12}	0.072 (0.029)	0.084 (0.034)	0.113 (0.051)
α	0.059 (0.012)	0.047 (0.012)	0.025 (0.008)
β	0.858 (0.042)	0.849 (0.050)	0.856 (0.057)
$\omega_{ii} = 1 - \alpha - \beta$	0.083 (0.033)	0.104 (0.042)	0.119 (0.053)
Correlation forecast	0.652 (0.009)	0.680 (0.021)	0.750 (0.017)
Beta forecast	1.556 (0.042)	1.397 (0.061)	1.147 (0.044)
MES	0.042 (0.0004)	0.036 (0.001)	0.028 (0.001)
LRMES	0.548 (0.010)	0.510 (0.015)	0.443 (0.012)
SRISK $\times 10^{10}$	10.582 (0.112)	8.159 (0.163)	8.678 (0.190)

Notes: Data for BAC, CIT, JPM and S&P500 index for the period 01/04/2001-12/31/2012. All coefficients, forecasts of correlation, beta, MES, LRMES and SRISK are reported at posterior means and standard deviations are given in brackets. MBIC is the Modified Bayesian Information Criterion.

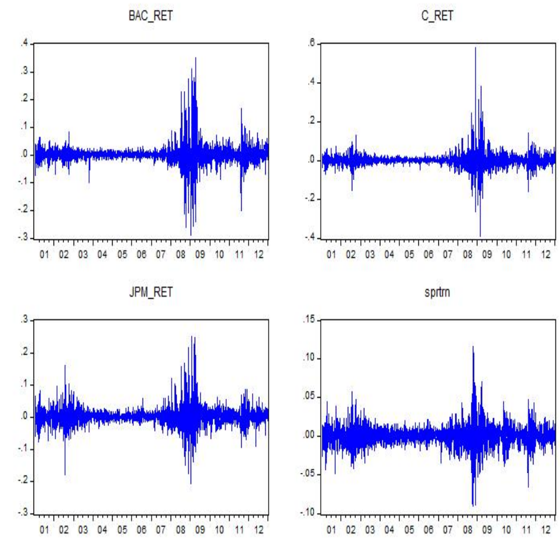

Figure 1: Returns: BAC, CIT, JPM, SPX

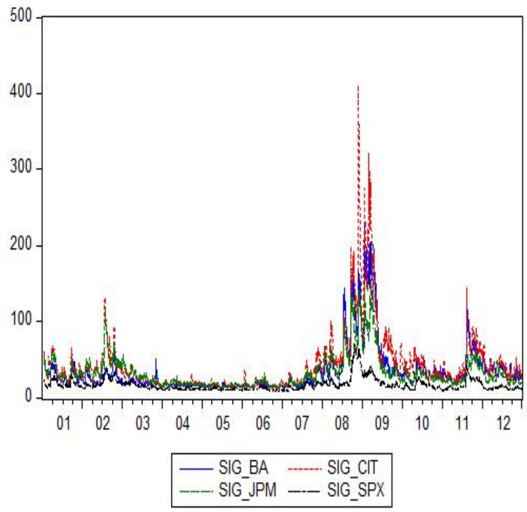

Figure 2: Annualized Volatility (GJR-GARCH): BAC,CIT, JPM, SPX

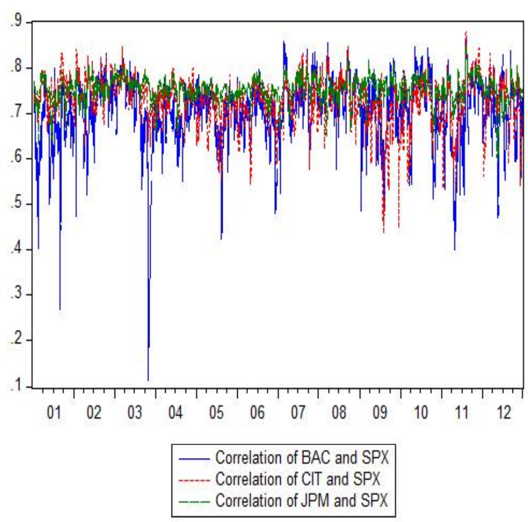

Figure 3: Dynamic correlation with the market (DCC-GJR-GARCH): BAC,CIT, JPM

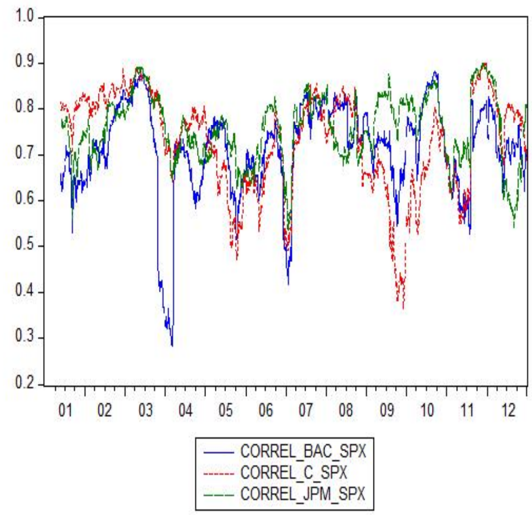

Figure 4: 100 day rolling correlation with the market : BAC,CIT, JPM

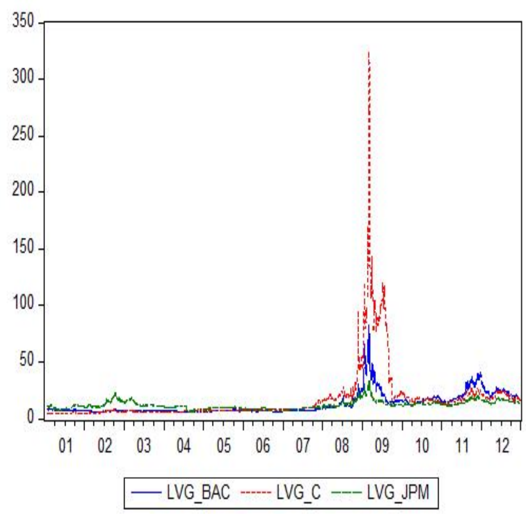

Figure 5: Leverage: BAC, CIT, JPM

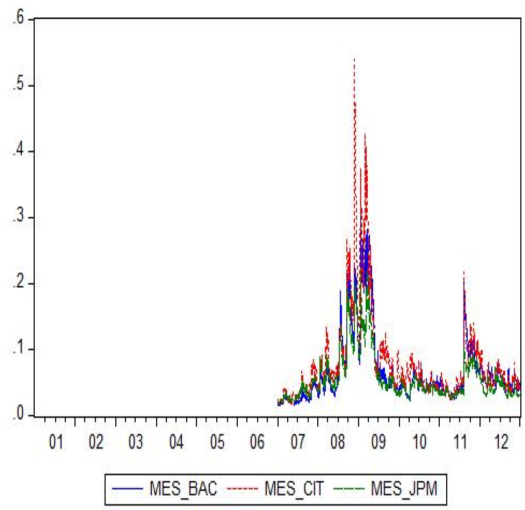

Figure 6: Marginal Expected Shortfall (MES) based on TARCH model: BAC,CIT, JPM

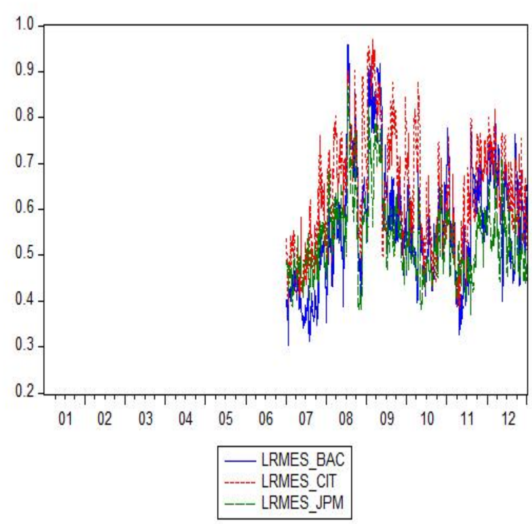

Figure 7: Long Run Marginal Expected Shortfall (LRMES) based on TARCH model: BAC,CIT, JPM

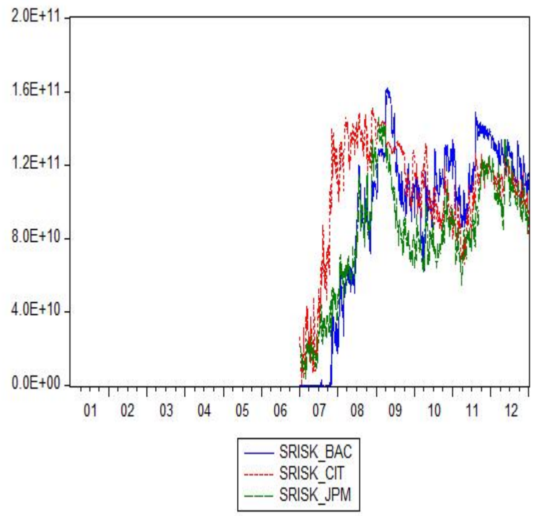

Figure 8: SRISK based on GTARCH model: BAC,CIT, JPM

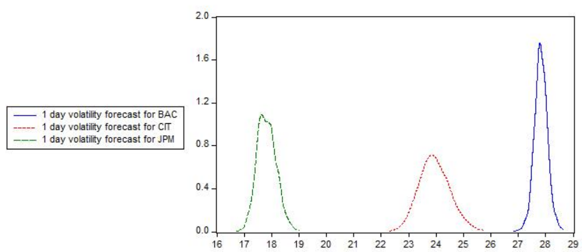

Figure 9: PDFs of one day forecasts of volatilty using TARCH model: BAC,C,JPM (2012/12/31)

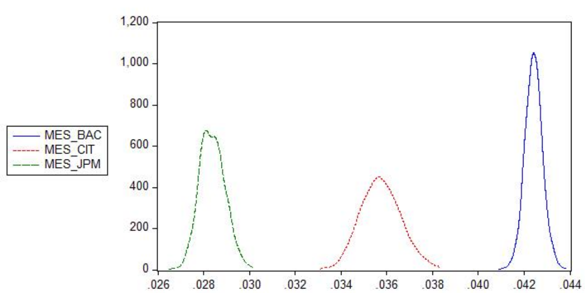

Figure 10: PDFs of Marginal Expected Shortfall in the period of low volatility: BAC,CIT, JPM (2012/12/31)

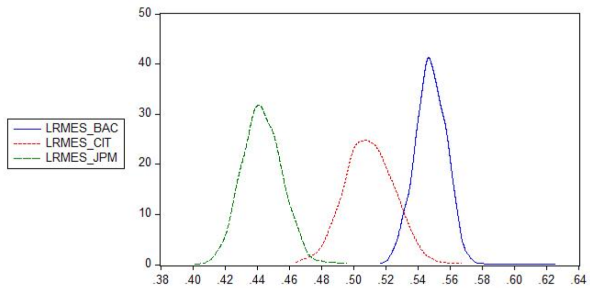

Figure 11: PDFs of Long Run Marginal Expected Shortfall in the period of low volatility: BAC,C,JPM (2012/12/31)

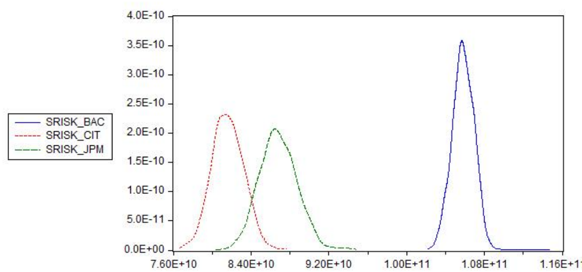

Figure 12: PDFs of SRISK in the period of low volatility: BAC,C,JPM (2012/12/31)

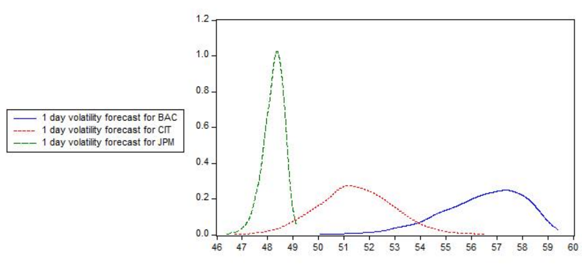

Figure 13: PDFs of one day forecasts of volatilty using TARCH model: BAC,CIT, JPM (2008/08/29)

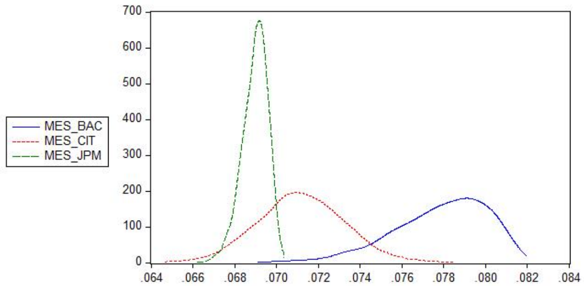

Figure 14: PDFs of Marginal Expected Shortfall in the period of high volatility: BAC,C,JPM (2008/08/29)

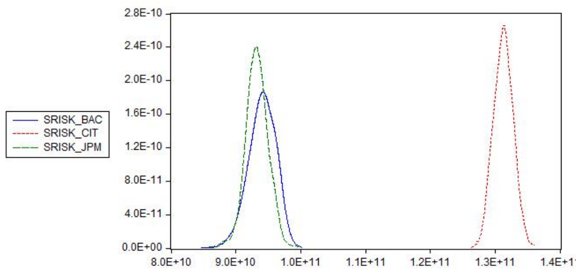

Figure 15: PDFs of SRISK in the period of high volatility: BAC,CIT, JPM (2008/08/29)

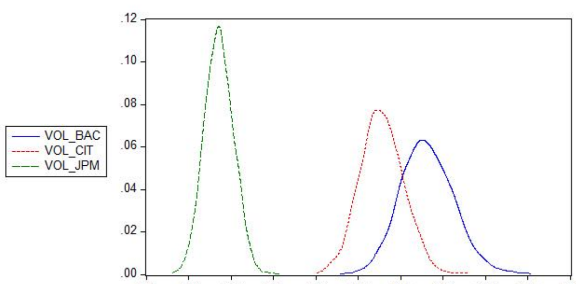

Figure 16: PDFs of one day forecasts of volatilty using TARCH model: BAC,CIT, JPM (2009/03/31)

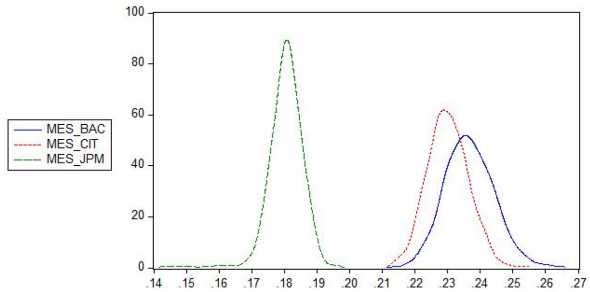

Figure 17: PDFs of Marginal Expected Shortfall in the period of high volatility: BAC,CIT, JPM (2009/03/31)

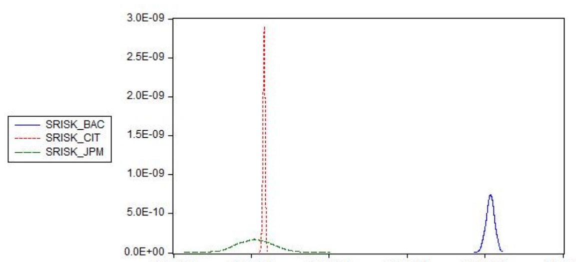

Figure 18: PDFs of SRISK in the period of high volatility: BAC,C,JPM (2009/03/31)