Bayesian Analysis of Systemic Risks Distributions

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Abstract

I propose Bayesian approach for estimation of systemic risks measures. The paper illustrates Markov Chain Monte Carlo (MCMC) estimation of MES (marginal expected shortfall), LRMES (long run marginal expected shortfall) and SRISK (the expected capital shortage of a firm conditional on a substantial market decline). The analysis is performed using the dynamic conditional correlations (DCC) model with asymmetric GJR-GARCH volatility and a generalized threshold conditional volatility model (GTARCH) that allows ARCH and GARCH parameters to change when returns innovations are negative. Using equity returns and credit default swap (CDS) spreads of large US banks I find that the proposed more general asymmetric volatility model has better fit, higher persistence of negative news and higher degree of risk aversion. Overall, I find that after accounting for uncertainty of parameters systemic risk distributions for large financial institutions could be highly overlapping, especially, during periods of moderate and high volatility. The SRISK measure is then extended to include equity illiquidity component. I advocate usage of distributions rather than point estimates of systemic risk measures by researchers and regulators.

KEY WORDS: Markov Chain Monte Carlo; Systemic Risk; Generalized Threshold Conditional Volatility; Dynamic Conditional Correlation; Illiquidity.

1 Introduction

After the 2007-2009 financial crisis the topic of systemic risk (or ”too big to fail”) became increasingly important in academic research and public policy discussion. Many papers introduced various systemic risk measures for large financial institutions based on publicly available market data as well as non-public firm specific data that can be accessed by regulators. Some measures uses daily and intra-daily equity and derivates prices, quarterly

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financial statements of publicly traded companies and macroeconomics data. In addition to that regulators use detailed financial positions of interconnectedness and complexity of the firm among other metrics. Bisias et al (2012), Acharya et al (2014), Arnold et al (2012), Brunnermeier and Oehmke (2013), and Benoit et al (2017) provide extensive reviews of various macroprudential and microprudential tests. In this paper I only consider the market based measures of systemic risks.

Acharya et al (2017) showed that if a financial firm is large and highly interconnected failure of such firm causes considerable strain to the rest of the financial sector and a negative externality on the rest of the economy. They propose systemic expected shortfall (SES) measure conditional on the system being undercapitalized. SES depends on the marginal expected shortfall (MES) and leverage of the firm. Brownlees and Engle (2017) generalize the measurement of SES by introducing dynamic conditional correlations (DCC) model with asymmetric GJR-GARCH volatility. They introduce the LRMES (long run marginal expected shortfall) and SRISK (the expected capital shortage of a firm conditional on a substantial market decline). SRISK is essentially a type of stress test that uses only market data and measures capital shortfall of a firm in case of a financial crisis with substantial market decline. SRISK depends on leverage, size, volatility and the conditional correlation of the firm with the market. The rankings for LRMES and SRISK for large global financial institutions introduced in this literature are weekly reported by the VLAB Volatility Institute[1]. However, this measures are reported without uncertainty around estimates and thus one cannot judge if the differences in rankings of large financial institutions are statistically significant. Brownlees and Engle (2017) introduced confidence intervals for the LRMES based on quantiles of 1-month simulated returns when the market is in distress. Their simulation procedure uses bootstrapped standardized residuals obtained from the point estimates of parameters without estimation errors. Thus parameter estimation errors are not incorporated in these intervals.[2] Since the simulation procedure is computationally intensive they also show how to estimate a simple approximation of LRMES without simulation. The LRMES and SRISK approximate measures are point estimates without uncertainty around them. Moreover, Brownlees and Engle (2017) do not analyze if individual financial institutions are actually different in terms of their systemic risks measures.

Many other popular market based measures were introduced in literature such as CoVaR (Adrian and Brunnrmeier (2016)), CoVaR with multivariate GARCH (Girardi and Ergun(2013)), systemic risk index CAITFIN (Allen et. al (2012), probability of default measures based on credit default swaps (CDS) (Huang et. al 2011). These studies looked at contribution of a firm in distress to overall risk of the financial system and also do not show how to measure and incorporate uncertainty and confidence intervals for systemic risk measures.

CDS spreads are widely used to assess default risks of financial institutions. The literature analyzing risks implied by CDS is growing. For example, Hull et. al (2004) among others studied the relation between CDS spreads, bond yield spreads and credit rating

[1] See http://vlab.stern.nyu.edu

[2] Appendix in Brownlees and Engle (2017) shows the simulation procedure. For comparison I also perform the same simulation procedure for 1 month and report results in Table 10.

announcements. Carr and Wu (2011) show the relation between CDS spreads and out-of-the-money American put options. The CDS premiums change dramatically over time and similar to equities exhibits volatility clustering. Oh and Patton (2017) study the systemic risk using dynamic copula model of CDS spreads where they use asymmetric GJR-GARCH volatility model with "bad news" resulting from positive residual. For modeling equity volatility the GJR-GARCH model is widely used with negative residual indicating "bad news".

If the systemic risk scores could be measured reasonably accurately it could benefit regulators when they set capital surcharges on financial institutions that have high contribution to the system-wide risk. The Financial Stability Board (FSB) publishes list of global systemically important banks (G-SIBs) allocated to buckets corresponding to required levels of additional capital buffers.[3] Their scoring methodology developed by the Basel Committee on Banking Supervision (BCBS) is based on equal weighted index of 5 indicators including size, interconnectedness, substitutability, complexity and cross-jurisdictional activity. The Office of Financial research (OFR) as well publishes brief series where they assess the relative riskiness of the G-SIBs by assigning relative scores on a similar number of attributes. Such scoring methods do not incorporate uncertainty of the estimated components and may result in inaccurate grouping of banks into buckets and corresponding levels of required capital buffers.[4]

In order to account for uncertainty in parameter estimation and provide distributions and confidence intervals for systemic risk measures the present paper shows how to estimate MES, LRMES and SRISK using Bayesian Markov Chain Monte Carlo (MCMC) algorithms.[5] In this paper I develop MCMC algorithms for estimation of asymmetric volatility models, correlation and systemic risk measures. The advantage of MCMC algorithms is their natural ability to generate posterior predictive densities for variables of interest, such as volatility, correlation and tail risks. I use Metropolis-Hastings steps with random walk draws. The algorithm is based on MCMC for threshold time series models in Goldman and Agbeyegbe (2007).

First, I introduce a generalized threshold conditional volatility model (GTARCH) and compare it to traditional asymmetric models of volatility. Since introduction of the generalized autoregressive conditional heteroscedasticity (GARCH) model there have been many extensions of GARCH models that resulted in better statistical fit and forecasts. For example, GJR-GARCH (Glosten, Jagannathan, & Runkle (1993)) is one of well-known extensions of GARCH models with an asymmetric term which captures the effect of negative shocks in equity prices on volatility commonly referred to as a "leverage" effect. The widely used asymmetric GJR-GARCH model (also known as TARCH) has a problem that the unconstrained estimated coefficient of α often has a negative value for equity indices. The typical solution to this problem is constrained estimation with positivity requirement making coef-

[3] http://www.fsb.org/wp-content/uploads/2016-list-of-global-systemically-important-banks-G-SIBs.pdf

[4] There are 5 buckets used by Basel methodology with the highest level of additional capital surcharge equal to 3.5% and the lowest equal to 1%.

[5] MCMC methods could be used for other measures as well. This paper focuses on the above three measures and volatility modeling.

ficient alpha close to zero. In the proposed more flexible GTARCH model both coefficients, ARCH (α) and GARCH (β), are allowed to change to reflect the asymmetry of volatility due to negative shocks. The GTARCH model better captures asymmetry in both ARCH and GARCH terms and allows for different levels of persistence in the regimes of positive and negative returns. In particular, GJR-GARCH model is a subset of this model with asymmetry only in ARCH; another subset is a simple GARCH model with no asymmetry. Other model introduced in the paper is the GTARCH0 model that allows for asymmetry only in GARCH.

Using MCMC method I estimate GTARCH family models for the logarithmic returns of equities and the log-differences of CDS spreads. For the analysis of equities I used data for Bank of America (BAC), JP Morgan Chase (JPM), Citi group (CIT) and S&P 500 between 2001-2012. I find that the most general GTARCH model fits better, shows higher persistence for negative news and lower persistence for positive news. The GTARCH model also shows higher risk aversion compared to other asymmetric GARCH models. Moreover, the suggested more flexible GTARCH model eliminates negative α bias that is typically found in the GJR-GARCH model for equity indices.[6] As for the CDS data of BAC and JPM I find that the GATRCH0 model has the best fit.

Second, I develop MCMC algorithm for estimation of the DCC (dynamic conditional correlations model) and find posterior distributions of correlation, beta, MES, LRMES and SRISK. In order to estimate the model Brownlees and Engle (2017) used two stage Maximum Likelihood estimation. In the first step they estimated GJR-GARCH volatility models for market and firm returns, while in the second stage they used standardized residuals to estimate the parameters of the dynamic conditional correlation (DCC) model. In order to simplify and speed up MCMC algorithms I perform estimation in similar two-steps, first estimating volatilities and then correlations.

It turns out that in a period of low volatility (2012/12/31) the SRISK measures are statistically different with 95% Highest Posterior Density Intervals (HPDIs) not overlapping for BAC, CIT and JPM, thus, they can be allocated in three separate buckets. Similar result holds for MES but not for LRMES. Since distributions of LRMES overlap for two banks the difference in their SRISK is driven by the difference in leverage. During periods of moderate (2008/08/29) or high volatility (2009/03/31) the 95% HPDIs of SRISK and other are measures are intersecting. SRISK ranks are mostly sensitive to a combination of leverage and size since often LRMES are not ranked differently with 95% highest posterior density intervals intersecting in various periods including low volatility period. Thus SRISK rankings of banks are justified distinguishing firms in terms of severity of the systemic risks they impose on the system when the leverage and size of the companies are substantially different.

Finally, I performed sensitivity analysis using equity illiquidity adjustment and simulation method for LRMES. The results are similar with illiquidity adjustment without simu-

[6]For S&P500 GJR-GARCH model unconstrained estimation results in a statistically significant negative α. For the GTARCH model α is not statistically significant.

lations of LRMES. The standard errors increase about ten times using simulation method and as a result the rankings become even less distinguishable.

Overall, this paper offers the following contributions. First, I propose a GTARCH model and compare its performance with subset volatility models commonly used in the literature for equities and log changes of CDS spreads. I show Bayesian MCMC estimation of GTARCH family models including popular GJR-GARCH and then extend it to Dynamic Conditional Correlation (DCC) model. Second, the equity volatility forecasts combined with correlation with the market are used for the measurement of systemic risks, MES, LRMES and SRISK, in a fashion similar to Acharya et al (2017) and Brownlees and Engle (2017) but incorporating uncertainty for risk measures.

The remainder of the paper is organized as follows. Section 2 presents the generalized threshold conditional volatility model (GTARCH). In Section 3 I review measures of systemic risks. Section 4 explains estimation of the GTARCH-DCC model, while Appendix provides details of the MCMC algorithms. Section 5 presents data analysis for Bank of America (BAC), JP Morgan Chase (JPM), Citi group (CIT) and S&P 500. Section 6 shows distribution of systemic risk measures in three cases for the periods of high, medium and low volatility. Section 7 presents conclusion and further work.

2 Generalized Threshold GARCH (GTARCH) model

A popular threshold ARCH or GJR-GARCH model (Glosten, Jagannathan, & Runkle (1993)) is an asymmetric volatility model which captures the impact of a negative news in equity returns also referred to a "leverage" effect. The model captures risk-aversion of investors with next day volatility being higher as a result of a negative news compared to the same positive news.

Consider a time series of logarithmic returns r_t with constant mean μ and the GJR-GARCH conditional variance σ_t^2 given by

$$\begin{aligned} r_t &= \mu + \epsilon_t \\ \sigma_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \gamma\epsilon_{t-1}^2 I_{(\epsilon_{t-1}<0)} + \beta\sigma_{t-1}^2 \end{aligned} \tag{1}$$

where ϵ_t is Gaussian (or other distribution) random variable, $I(\epsilon_{t-1} < 0)$ is a dummy variable equal to one when previous day innovation ϵ_{t-1} is negative; α and β are ARCH and GARCH parameters, and γ is an asymmetric term capturing risk aversion.

Alternatively, this model can be written as a two-regime model for the ARCH term:

$$\sigma_t^2 = \omega + \alpha_1\epsilon_{t-1}^2 I_{(\epsilon_{t-1}<0)} + \alpha_2\epsilon_{t-1}^2 I_{(\epsilon_{t-1}\geq 0)} + \beta\sigma_{t-1}^2 \tag{2}$$

The properties of risk-aversion for equity returns imply that $\alpha_1 \geq \alpha_2$ or $\gamma = \alpha_1 - \alpha_2 \geq 0$.

The widely used asymmetric GJR-GARCH model has a problem that the unconstrained estimated coefficient of positive news $\alpha = \alpha_2$ often has a meaningless negative value for equity indices. A constrained optimization imposing positivity on all variance parameters results in α slightly positive and very close to zero.

The Generalized Threshold GARCH (GTARCH) model that I introduce allows both ARCH and GARCH terms to reflect asymmetry of volatility due to negative news.

$$\sigma_t^2 = \omega + \alpha\epsilon_t^2 + \gamma\epsilon_t^2 I(\epsilon_{t-1} < 0) + \beta\sigma_{t-1}^2 + \delta\sigma_{t-1}^2 I(\epsilon_{t-1} < 0), \qquad (3)$$

where added term δ reflects degree of asymmetric response in the GARCH term. In this model both parameters γ and δ create the asymmetric response of volatility to negative shocks.

The GTARCH model can be also written as a two-regime threshold model:

$$\sigma_t^2 = \omega + \alpha_1\epsilon_{t-1}^2 I_{(\epsilon_{t-1}<0)} + \alpha_2\epsilon_{t-1}^2 I_{(\epsilon_{t-1}\geq 0)} + \beta_1\sigma_{t-1}^2 I_{(\epsilon_{t-1}<0)} + \beta_2\sigma_{t-1}^2 I_{(\epsilon_{t-1}\geq 0)} \qquad (4)$$

The more general GTARCH model due to its flexibility of parameters shows different dynamics for GARCH parameters when the news is negative and allows for higher persistence in the regime of negative news. This in turn takes away the negative bias from α_2 which measures the reaction to the positive news. At the same time estimation of extra parameter β_2 is a straightforward extension as shown in Section 4.

Data estimation in Section 5 shows that allowing both ARCH and GARCH parameters to change with negative news results in better statistical fit. Moreover, the GTARCH model not only better captures the asymmetric effect but also shows higher persistence for negative returns compared to its subset GJR-GARCH model. In addition the coefficients of μ and ω could be allowed to change with regime to make the model even more flexible.[7]

In this paper I consider the following GTARCH family models with constraints imposed on GTARCH parameters:

GTARCH Family Models	Constraint
GTARCH	None
GJR-GARCH (TARCH)	$\beta_1 = \beta_2$
GTARCH0	$\alpha_1 = \alpha_2$
GARCH	$\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$

The GTARCH includes a family of models with and without asymmetry. As a special case it allows for GJR-GARCH, GTARCH0 and GARCH. For example, in a model that I call GTARCH0 the asymmetric effect is only in the GARCH term.

2.1 Stationarity of the GTARCH Model

The weak stationarity condition in the GARCH model and the long run unconditional variance σ^2 are given by the following equations:

[7] However, interpretation of risk aversion is less intuitive. I estimated such models for sensitivity and results are available on request.

$$\alpha + \beta < 1, \quad \sigma^2 = \frac{\omega}{1-\alpha-\beta}$$

For the GTARCH model let $\theta = E(I(\epsilon_t < 0))$ which is expected percentage of observations with negative news that should be in theory equal to 1/2. Then the weak stationarity condition and the unconditional variance are given by

$$(\alpha_1+\beta_1)\theta+(\alpha_2+\beta_2)(1-\theta) = (\alpha_1+\alpha_2+\beta_1+\beta_2)/2 < 1, \quad \sigma^2 = \frac{\omega}{1-(\alpha_1+\alpha_2+\beta_1+\beta_2)/2}$$

For actual data applications stationarity conditions are typically incorporated as estimation constraints.

3 Systemic Risk Measures

Brownlees and Engle (2017) introduced several ways to measure systemic risks with and without simulations. They use the Dynamic Conditional Correlations (DCC) model of Engle (2002,2009) for the logarithmic returns of the firm r_t and the market $r_{m,t}$. The conditional variances are modelled using the GJR-GARCH model. Compared to their model I add a constant term μ for each asset as well as GTARCH family models introduced in Section 2.

The returns can be written as

$$\begin{aligned} r_{mt} &= \mu_m + \sigma_{mt}\epsilon_{mt} \\ r_t &= \mu_i + \sigma_t\rho_t\epsilon_{mt} + \sigma_t\sqrt{1-\rho_t^2}\epsilon_t \end{aligned} \tag{5}$$

where $\epsilon_{mt}, \epsilon_t$ are independent and identically distributed variables with zero means and unit variances, σ_t and σ_{mt} are conditional standard deviations of the firm return and the market return correspondingly, and ρ_t is conditional correlation between the firm and the market. This model can be also written as the dynamic conditional beta model with $\beta_t = \rho_t\frac{\sigma_t}{\sigma_{mt}}$ and tail dependence on correlation of firm returns and the market.

The first considered systemic risk measure is the daily marginal expected shortfall (MES) which is the conditional expectation of a daily return of a financial institution given that the market return falls below threshold level C. Acharya et al (2017) considered the threshold equal to the 5% quantile for the market returns. In the original version of VLAB it was assumed that threshold $C = -2\%$ and I use the same estimate in the analysis below.[8] Other values for C using various quantiles could be explored.

$$\begin{aligned} MES_{t-1} &= E_{t-1}(r_t|r_{mt} < C) \\ &= \sigma_t\rho_tE_{t-1}(\epsilon_{mt}|\epsilon_{mt} \le C/\sigma_{mt}) + \sigma_t\sqrt{1-\rho_t^2}E_{t-1}(\epsilon_t|\epsilon_{mt} \le C/\sigma_{mt}) \end{aligned} \tag{6}$$

[8]Current version of VLAB lists only LRMES threshold but not MES.

The computation of the expected shortfall is perfomed using nonparametric estimates of Scaillet (2005):

$$E_{t-1}(e_{mt}|e_{mt} \leq \alpha) = \frac{\sum_{i=1}^{t-1} e_{mi}\Phi_h(\frac{\alpha - e_{mi}}{h})}{\sum_{i=1}^{t-1} \Phi_h(\frac{\alpha - e_{mi}}{h})}$$

where $\alpha = C/\sigma_{mt}$, $\Phi_h(t) = \int_{-\infty}^{t/h} \phi(u)du$, $\phi(u)$ is a standard normal probability distribution function used as kernel, and $h = T^{-1/5}$ is the bandwidth parameter. The nonparamteric method above allows more smooth estimate of MES since the number of points in the tail for computation of expected shortfall is limitted.

The second measure is the long run marginal expected shortfall LRMES based on the expectation of the cumulative h month firm return conditioned on the event that the market falls by more than d% in six months. VLAB uses $h = 6$ months and allows choice of d which is by default equal to 40% for the US financial firms. Brownlees and Engle (2017) set $h = 1$ month with threshold $d = 10\%$.[9]

First, I find LRMES meausure without simulation using the approximation below following the methodology in VLAB.[10] The crisis threshold for the S&P 500 Composite Index decline is set to $d = 40\%$ which is consistent with analysis with simulations in VLAB.

$$LRMES_t = 1 - exp(ln(1-d) * \beta_t) \tag{7}$$

Second, a simulation procedure is used to obtain LRMES using bootstrapped standardized residuals from equation (5) where I obtain forecasts of variances and correlations from the posterior mean estimates of parameters while in Brownlees and Engle (20017) they are point Maximum Likelihood estimates. As in Brownlees and Engle (2017) LRMES is computed as empirical average of firm cumulative $h = 1$ month returns conditional on the market fall more than $d = 10\%$.

Finally, LRMES is combined with firm equity and debt in order to compute the capital shortfall of the firm based on the potential capital loss in h months:

$$SRISK_t = max\{0; kD_t - (1-k)(1 - LRMES_t)E_t\} \tag{8}$$

where D_t is the book value of Debt at time t, E_t is the market value of equity at time t and $k = 8\%$ is the prudential capital ratio of the US banks.[11] It is assumed that the capital loss happens only due to the loss in the market capitalization $LRMES * E_t$.

The VLAB rankings for SRISK and components are presented in Table 1. Bank of America (BAC), Citigroup (CIT) and JP Morgan Chase (JPM) are ranked as the top three highest systemically important US financial firms for the period August 29, 2008 - December 31, 2012. In terms of SRISK % the three banks above account for 40-50% of the whole list of

[9] For sensitivity threshold equal to 20% is also considered and high correlation in resulting SRISK meausures for large firms is found.

[10] I use VLAB to compare estimates of SRISK in Section 5.

[11] Brownlees and Engle (2017) for sensitivity also set $k = 10\%$ and found high correlation in resulting SRISK measures for large firms.

the US financial institutions. The first panel in Table 1 shows fourteen highest systemically important financial institutions using SRISK about two weeks before the collapse of Lehman Brothers. Interestingly, Lehman Brothers is ranked eleventh. However, using LRMES it would be ranked third on the list following Fannie Mae and Freddie Mac. Below I analyze properties of MES, LRMES and SRISK as they may potentially give different rankings.

Table 1 here

In order to check for robustness of SRISK I construct a modified SRISK adjusted for equity illiquidity. My extension is based on liquidity adjusted Value at Risk (VaR) following Bangia et al. (1999) among others. I compute the relative spread based on bid-ask closing prices and mid price as $S = \frac{ASK-BID}{Mid}$. Then returns are adjusted by spread ($r^* = r - S/2$) assuming that half of the spread accounts for transaction costs. Here the return risk and liquidity risk are modeled jointly. Next, I use modified lower equity returns r^* for the computation of systemic risk measures. Since in SRISK debt level is assumed to be not changing for h months ahead fI only perform illiquidity adjustment for equity.

4 Estimation of the GTARCH-DCC Model

I use Bayesian Markov Chain Monte Carlo (MCMC) algorithms to estimate posterior distributions of parameters of the GTARCH and DCC model. MCMC are especially useful when the dimension of parameters is high, since the problems of multiple maxima or of initial starting values are avoided. A simple intuitive explanation of the Metropolis-Hastings algorithm is given in Chib and Greenberg (1995). Goldman and Agbeyegbe (2007) developed MCMC algorithm for the estimation of a general class of multiple threshold time series of the U.S. short term interest rates. Their model nests the threshold autoregressive model (TAR or SETAR), ARMA, GARCH, and CKLS models. The algorithm allows to estimate jointly parameters of all regimes as well as threshold parameters. Goldman, Nam, Tsurumi and Wang (2013) extended the MCMC algorithm for the fractional integration parameter and estimated threshold fractionally integrated (TARFIMA) for realized volatilities of intraday ETF and stock returns.

In this paper I use Metropolis-Hastings steps with random walk draws for the GTARCH model in equation (4) based on MCMC for ARMA-GARCH models of Goldman and Tsurumi (2005) and threshold models of Goldman and Agbeyegbe (2007). The GTARCH model is a special case when there are two regimes for the GARCH model determined by ϵ_{t-1} with threshold equal to 0. In the simpler and computationally faster algorithm I estimate the regression parameter and GARCH parameters for two regimes: ($\epsilon_{t-1} < 0$ and $\epsilon_{t-1} \geq 0$).

After the parameters of the GTARCH model as well as all nested models including GJR-GARCH, GTARCH0 and GARCH are estimated, I find conditional variances over time and standardized residuals given by $z_{i,t} = \dfrac{r_{i,t} - \mu_i}{\sigma_{it}}$ and $z_{m,t} = \dfrac{r_{m,t} - \mu_m}{\sigma_{mt}}$, where $r_{i,t}$ and $r_{m,t}$ are daily logarithmic returns of firm i and the market correspondingly.

Next, I estimate the parameters α_{Ci} and β_{Ci} of the dynamic correlation $\rho_{i,t}$ for the standardized residuals using the DCC model of Engle (2002):

$$
\begin{aligned}
q_{im,t} &= \omega_{im}(1-\alpha_{Ci}-\beta_{Ci}) + \alpha_{Ci} z_{i,t} z_{m,t} + \beta_{Ci} q_{im,t-1} \\
\rho_{i,t} &= \frac{q_{im,t}}{\sqrt{q_{ii,t}\; q_{mm,t}}}
\end{aligned}
\tag{9}
$$

where ω_{im} is the unconditional covariance of $z_{i,t}$ and $z_{m,t}$ and

$$
Q_{im,t} = \begin{bmatrix} q_{ii,t} & q_{im,t} \\ q_{im,t} & q_{mm,t} \end{bmatrix}
$$

is a positive-definite pseudo-correlation matrix that needs to be re-normalized to get correlation at each point. Tse and Tsui (2002) proposed an alternative DCC model for the correlation with smoothing that typically results in less volatile correlation. That model involves choice of smoothing parameter and does not need normalization of the Q.

In order to estimate the DCC model in equation (9) Brownlees and Engle (2017) used two stage Maximum Likelihood estimation. In the first step they estimated GJR-GARCH volatility models for market and firm returns, while in the second stage they used standardized residuals to estimate the parameters of the dynamic conditional correlation (DCC) model. In order to simplify and speed up MCMC algorithms I perform estimation in similar two-steps, first estimating volatilities and then correlations. Details of the MCMC algorithms and model selection criteria are given in Appendix.

5 Data Analysis

In this section I estimate volatilities, correlations and systemic risks for Bank of America (BAC), Citigroup (CIT) and JP Morgan Chase (JPM) ranked by VLAB as the top three systemically important US financial firms for the period August 29, 2008 - December 31, 2012 shown in Table 1.[12] I use the equity and debt data are for the period 1/04/2001-12/31/2012 similar to Brownlees and Engle (2017) whose sample ends in 2012. I use market data on stock prices, market capitalization, book value of debt and also credit default swap (CDS) spreads. The data for equity prices, returns and market capitalization are from CRSP. The book value of debt is from COMPUSTAT. The CDS data for five year secured bonds are from Bloomberg.

The equity returns daily data are presented in Figure 1. All equity returns exhibit similar volatility clustering between 2008-2009 and in August 2011. Table 2 shows the summary statistics where all the series have large kurtosis over 10, some skewness and small autocorrelation.[13]

Figure 1 and Table 2 here

[12] Since the same methodology can be applied for the rest of the financial sector I focus just on three major banks for illustration of estimation and results of capital shortfall under market stress.

[13] I did not use the autoregressive term in the return equation as it was never significant.

Table 3 shows the results of MCMC estimation of GTARCH family volatility models for SPX. I estimate GTARCH with all parameters $(\alpha_1, \alpha_2, \beta_1, \beta_2)$, GJR-GARCH $(\beta_1 = \beta_2)$, GTARCH0 $(\alpha_1 = \alpha_2)$ and GARCH ($\alpha_1 = \alpha_2, \beta_1 = \beta_2$). For each model the posterior mean and standard deviation of parameters are reported. In the Bayesian approach it is common to use the highest posterior density intervals (HPDI) for hypothesis testing. I report 95% HPDI for the one day forecast of volatility which can be compared for all models and discuss confidence intervals for GARCH parameters below.

First, I estimated GTARCH models without constraints on positivity of α and the 95% HPDI for α_2 is [-0.026, 0.005]; thus α_2 is not statistically different from zero in the GTARCH model. However, when I estimate the GJR-GARCH model without positivity constraint on α it results in statistically significant negative posterior mean of $\alpha_2 = -0.021$ with 95% HPDI equal to [-0.034,-0.007]. The interpretation of negative α_2 that positive news reduces volatility in the next period is unintuitive. The reason for negative α_2 is that the GJR-GARCH model is not flexible enough to allow change in regime and persistence of GARCH parameters other than in α. This creates a bias in α that captures change in regime for other parameters.[14] Since GARCH parameters need to be positive I impose constraints in MCMC procedure which results in estimated α_2 being positive but very close to zero in these models.

The most general GTARCH volatility model is selected using minimum of the MBIC information criterion[15] either evaluated at the posterior mean or mode of the parameters. All models satisfy stationarity condition with overall $persistence = (\alpha_1+\alpha_2+\beta_1+\beta_2)/2 < 1$. Clearly in the GTARCH model both α and β change with regime showing the asymmetric effect present in both ARCH and GARCH terms and higher persistence in the regime of negative news. For example, the 95% HPDI for β_1 is [0.945, 1.001] and for β_2 is [0.810, 0.857]. This shows different β for two regimes as the 95% intervals do not cross. Similarly α is different for two regimes with 95% HPDI for α_1 equal to [0.134, 0.188] and for α_2 [0.000, 0.012]. The GJR-GARCH model that allows only the coefficient of α to change with regime has the 95% HPDIs equal to [0.152, 0.222] and [0.000,0.008] for α_1 and α_2 correspondingly.

Table 3 also shows the degree of risk aversion in each model measured by the correlation between returns r_{t-1} and log difference of fitted conditional variance $log(\sigma_t^2/\sigma_{t-1}^2)$ for each model. The more negative correlation implies higher degree of risk aversion because of asymmetrically higher volatility for negative returns. For comparison the log difference in VIX index has correlation with the S&P 500 return of about -0.7. Table 3 shows that the highest degree of risk aversion is captured by the GTARCH model and the smallest correlation is for the symmetrical GARCH model.

Table 3 here

[14] For sensitivity analysis I estimated a more general GTARCH model. In addition to α and β I also allowed the constant terms μ and ω to change with regime for negative returns. The latter model resulted in very small positive α_2; without imposing positivity constraint the 95% HPDI is [0.000, 0.006]. So α_2 is indeed close to zero.

[15] MBIC is the Modified Bayesian Information Criterion explained in the Appendix.

The dynamic volatilities resulting from GTARCH family models at posterior means of parameters are plotted in Figure 2. The plot of annualized volatility forecast $\sqrt{252}\sigma_t$ for SPX shows the highest level during the financial crisis and the second peak in August 2011. All four models give similar dynamics but in order to compare volatility estimates at any particular time I plot kernel densities of the volatility distributions on Figures 3-5 rather than only comparing posterior means.

Figure 2 here

The volatility distributions estimated at the end of the sample show some overlap. Figure 3A shows posterior distributions of one-day SPX volatility forecasts on 2012/12/31 when volatility level was low. The 95% HPDIs presented earlier in Table 3 show that the one day forecasts for GTARCH and GTARCH0 overlap. Also GJR-GARCH and GARCH forecasts overlap. In this case GTARCH model produces higher forecast of volatility than GJR-GARCH (TARCH) and GARCH models.

Figure 3B shows forecasts at a time of moderate level of volatility on 2008/08/29 before Lehman's collapse. Again distributions for GTARCH and GTARCH0 overlap and distributions for GJR-GARCH and GARCH also intersect. At this point GTARCH model produces lower forecast of volatility than GJR-GARCH and GARCH models.

Figure 3C presents similar distributions at a time of high volatility on 2009/03/31. At this point GTARCH and GJR-GARCH distributions are very close to each other producing similar forecasts. GARCH and GTARCH0 as well produce similar forecasts which are about 700 basis points higher than GTARCH and GJR-GARCH. Interestingly, the GTARCH model is somewhat less procyclical than GJR-GARCH (TARCH) and GARCH models: in Figures 3A-3C compared to other models GTARCH forecasts higher volatility when markets are calm and lower volatility when markets are in distress.

Figure 3 here

The dynamics of the GJR-GARCH volatility estimated at posterior means of parameters for three banks and SP500 index are presented in Figure 4. Figure 4A shows that volatility increased for all banks and the SPX following Lehman Brothers' bankruptcy filing on 9/15/2008. Specific events for other banks that lead to higher volatility include: JPM's acquisition of Bear Sterns backed by Government support on 3/16/2008; Bank of America's acquisition of Meryll Lynch on 9/15/2008. Major SCAP infusions were received by Bank of America and Citigroup on 10/13/08. CIT volatility is the highest among the group reaching the peak of over 350% in the midst of the 2008-2009 crisis. Before the financial crisis JPM had higher level of volatility than BAC, while during the crisis and afterwards BAC volatility level exceeded JPM. Not surprisingly the SPX has lower equity volatility than banks. For robustness check instead of showing very similar graphs over time with GTARCH and GARCH models I plotted a simple HIGH-LOW daily estimate of volatility for JPM and BAC in Figure 4B. High and low daily prices are from CRSP. This simple alternative non-parametric measure (HIGH-LOW) captures similar patterns in volatility as the GJR-GARCH model.

Figure 4 here

The posterior predictive distributions of annualized volatility forecast $\sqrt{252}\sigma_{T+1}$ using the GJR-GARCH model for three banks are shown in Figure 5. Panel A presents case 1 of low volatility (T=2012/12/31), panel B is for case 2 of medium volatility (T=2008/08/29) and panel C is for case 3 of high volatility (T=2009/3/31). In case 1 of low volatility the posterior distributions are distinct and volatility forecasts can be ranked from the highest to the lowest: 1. BAC, 2. CIT, 3. JPM. It is important that banks can be distinguished as volatility forecast is an essential part of MES which I consider in the next section. However, in the period of moderate volatility (2008/08/29) the forecasts distributions highly overlap and volatilities can't be ranked. Finally for 2009/03/31 BAC again can be ranked with the highest volatility followed by CIT and JPM. Table 7 column 2 shows 95 % HPDIs for volatility and confirms that while forecasts are statistically different for case 1 (low volatility) and case 3 (high volatility) they highly overlap in case 2 (medium volatility).

Figure 5 here

Figure 6A shows leverage ratios for three banks. The quasi market value of leverage is constructed using market value of equity and book value of debt: $LVG = (D_t + E_t)/E_t$, where daily data for market capitalization E_t is from CRSP and quarterly book value of debt is from COMPUSTAT. Before the financial crisis the leverage of JPM exceeded that of BAC and CIT. Starting from 2008 CIT leverage skyrocketed, followed by BAC. JPM had the least leverage among three largest banks during the crisis.

Figure 6 also shows the CDS spreads and log-differences of CDS spreads. The CDS weekly and daily spreads on the 5 year secured bonds were obtained from Bloomberg. I present weekly data for spreads in Figure 6B and log difference in spreads in Figure 6C. The CDS spreads for BAC, CIT and JPM seem to move together to some extent. Since the crisis CDS spreads were the highest for CIT, followed by BAC and by JPM. As with equity volatility the CDS spreads were higher for JPM before the financial crisis and lower since the crisis. The log-differences of CDS spreads exhibit volatility clustering similar to equity returns.

I consider the GTARCH models for the log-differences of CDS spreads. Unlike the equity returns the bad news in CDS market is when the spreads increase and one might expect higher persistence in regime 2. In Table 4 I present the results of fitting GTARCH family volatility models for log-differences in CDS spreads of JPM. Even though the CDS spreads typically have significant positive skewness the log-differences of CDS spreads do not show considerable skewness as can be seen from the summary statistics in Table 2. AR(1) coefficients are small as well. Interestingly the GTARCH0 model is selected among others using MBIC information criterion. Only asymmetry in the GARCH term (rather than the ARCH term in the traditional GJR-GARCH model) seems to be important for CDS volatility. CDS volatility was modelled using GJR-GARCH model by Oh and Patton (2017) . The proposed asymmetric GTARCH models may be also useful for modeling value at risk (VaR) for CDS. With Dodd Frank regulation on mandatory clearing setting initial margins using VaR models is a common practice where asymmetric volatility models can be applied.

Figure 6 and Table 4 here

Once the volatility models were estimated I find standardized residuals at posterior means of parameters and estimate the DCC model as explained in Section 4. The resulting dynamic conditional correlation of firms with the market using DCC-GJR-GARCH model is given in Figure 7A. For comparison I also present 100-day rolling correlations in Figure 6B. Both graphs show changing correlation over time with averages between 0.70-0.75; distributions of correlations are negatively skewed with longer left tail.[16]

Figure 7C shows the dynamics of beta over time with average between 1.35-1.54. The distribution of betas is highly skewed to the right with the maximum levels reaching 3.6-6.7. Interestingly, the beta for JPM reached higher maximum of 6.7 compared to BAC and CIT during 2008-2009, while JPM volatility was the lowest among three banks. Beta has essential contribution to LRMES with higher beta resulting in higher LRMES.

Figure 7 here

The posterior means and standard deviations of the correlation parameters in the DCC-GJR-GARCH model are given in Table 5.[17] I also present one day forecasts of correlation, beta, MES and six month forecasts of LRMES and SRISK. This forecasts are at the end of the sample (12/31/2012) when volatility was low. BAC has higher posterior mean of beta, MES, LRMES and SRISK than other two banks. The full posterior distributions of these risk measures are presented in Figures 9-11 and will be discussed in Section 6.

Table 6 presents various risk estimates of GTARCH family models for JPM. At the end of 2012 the Beta ranged between 1.00-1.15, MES 2.8%-3.0%, LRMES 40%-45% and SRISK 80-87 $billion for various models. MES estimates are very similar across models. GARCH posterior means are somewhat higher for Beta, LRMES and SRISK compared to other models, but the 95% HPDIs show that the differences are not statistically significant.

Tables 5 and 6 here

[16]Tse and Tsui (2002) proposed an alternative DCC model for the correlation with smoothing that typically results in less volatile correlation.

[17]To save space I did not present parameter estimates for other models since they were similar. I compare various risks for all models in Table 6.

The systemic risk measures MES, LRMES and SRISK over time are presented in Figures 8A, 8B and 8C correspondingly. All three measures are estimated at the posterior means of parameters of the DCC-GJR-GARCH model using equations (6)-(8).[18] These measures start in 2006 as I use half of the sample to estimate the shortfall for the first observation of MES. For MES at any point t I use all available data up to time t. Similar to volatility during the crisis the MES is the highest for CIT, followed by BAC and JPM. However, more careful analysis of the distribution of MES at a particular point is needed.

LRMES and SRISK also build up during the crisis and show considerable volatility afterwords. The SRISK posterior average values are similar to values reported by VLAB but could be different potentially due estimation period used in VLAB At the end of the sample (2012/12/31) SRISK using MCMC for the GJR-GARCH model reported in Table 5 is about 98.1 $ billion for BAC with standard deviation of 1.3, 72.2 $ billion for CIT with standard deviation of 1.4 and 83.7 $ billion for JPM with standard deviation of 1.5. The VLAB SRISK values do not have standard deviations and are reported in Table 1 ass 100.7 $ billion for BAC, 84.2 $ billion for CIT and 83.0 $ billion for JPM. There seems to be a difference for CIT while other banks have SRISK within 1 or 2 standard deviations from VLAB values.

Figure 8 here

6 Posterior Distributions of MES, LRMES and SRISK

Finally I present posterior predictive distributions of MES_{T+1}, $LRMES_{T+180}$ and $SRISK_{T+180}$ derived from MCMC draws in Figures 9, 10 and 11 correspondingly. As with volatility forecasts I consider three starting points for T. Panel A presents case 1 for low volatility (T=2012/12/31); panel B is for case 2 of medium volatility (T=2008/08/29) and panel C is for case 3 of high volatility (T=2009/3/31). The 95% HPDIs for volatility, beta, MES, LRMES and SRISK were presented in Table 7.

Table 7

I show the results with the GJR-GARCH model as in Brownlees and Engle (2017) even though the GTARCH model has better fit. Results below turned out to be not statistically different if GTARCH model is used. Figures 9-11 and Table 7 compare BAC, CIT and JPM in terms of the posterior pdfs of MES, LRMES and SRISK.

The results for MES in Figure 9 are similar to the results for volatility in Figure 5 with the difference that for the case 1 now distributions for JPM and CIT are more close and for case 3 BAC and CIT have slightly overlapping 95% HPDIs. As before case 3 shows high overlap between three distributions. Separate rankings for MES are possible only for the

[18] I follow Brownlees and Engle (2017) in using DCC-GJR-GARCH model. The dynamics using other models is similar.

period of low volatility where 95% HPDIs do not cross. For case 2 (2008/08/31) banks are identical and for case 3 (2009/03/31) only JPM can be ranked lower and other two banks are not distinguishable. Thus, MES does not seem to be a good measure to rank banks when volatility is high or even moderate. But it could be used to potentially assign the financial institutions for buckets of not highly overlapping distributions. This way there will be three separate buckets for case 1, one bucket for case 2 and two buckets for case 3.

Next I consider LRMES in Figure 10. For case 1 of low volatility (2012/12/31) posterior pdfs of LRMES of CIT and JPM are highly overlapping (as can be also seen from 95% HPDIs reported in Table 7). BAC has higher average LRMES and can be ranked separately. For cases 2 and 3 the distributions overlap heavily and these three firms can't be placed in separate buckets.

Finally, based on posterior pdfs of SRISK in Figure 11 and confidence intervals in Table 7 for the case 1 of low volatility (2012/12/31) banks can be ranked by SRISK as: #1 BAC (the highest), #2 JPM and #3 CIT. For the case 2 of medium volatility CIT has higher SRISK risk while BAC and JPM are the same. For case 3 of high volatility (2009/3/24) BAC has higher risk while JPM and CIT are the same. So three separate buckets are resulted for case 1 and two buckets for cases 2 and 3.

Figure 9 here

Figure 10 here

Figure 11 here

In Table 8 I analyze the components of SRISK for 3 cases above. SRISK is a function of (Leverage (+), Size (+), Beta (+)). Beta and SRISK are estimated at posterior means of DCC-GJR-GARCH models with standard deviations in brackets. For case 1 of low volatility (2012/12/31) as discussed above based on SRISK banks can be ranked as: #1 BAC (the highest), #2 JPM and #3 CIT. The reason that BAC has higher SRISK is due to higher leverage=17 and higher estimated Beta=1.285. CIT has the lowest assets and equity value. It is otherwise similar to JPM in terms of beta and leverage. Thus, CIT is ranked the lowest due its smaller size.

For the case 2 of medium volatility (2008/08/29) on the verge of the financial crisis posterior PDFs for SRISK indicated that CIT has higher risk while BAC and JPM are the same. At that time CIT had the highest leverage of 20 and the highest size measured by assets. CIT's equity value is smaller than for other two banks thus assets are higher due to leverage. CIT's beta is similar to JPM. Based on much higher leverage CIT has the highest SRISK as would be expected. JPM and BAC are similar for all measures except a bit higher beta for BAC.

For case 3 of high volatility (2009/3/31) posterior pdfs of SRISK indicate that BAC has higher risk while JPM and CIT are the same. BAC had leverage of 49 which is more than twice of JPM at a time. It also had much higher beta of 4.16, thus, BAC had significantly higher SRISK than JPM and they should be in different buckets in terms of severity of the systemic risks they impose on the system. Interestingly, CIT which leverage skyrocketed to 121 at the peak of the crisis is ranked similarly to JPM. Even though CIT also had higher Beta than JPM the result of similar SRISK is due to the depressed value of equity and assets ($14 Billion and $1697 correspondingly). Thus, CIT which was in the most urgent need of financial support would be considered in the same bucket as JPM. This simple example illustrates that SRISK may be influenced by the size of the firm more heavily compared to other measures such as beta and leverage.

Table 8

It turns out that SRISK measures are statistically different with distributions not crossing in case 1. This means that in the periods of low volatility the SRISK rankings of banks are justified distinguishing firms in terms of severity of the systemic risks they impose on the system. However, during periods of moderate or high volatility the rankings are intersecting. SRISK ranks are mostly sensitive to a combination of leverage and size since LRMES are not ranked differently with 95% highest posterior density intervals intersecting in periods of moderate and high volatility. [19]

6.1 SRISK with simulation

In this section similar to Brownlees and Engle (2017) I show the simulation based approach for computation of LRMES. I used 100,000 simulation paths of bootstrapped residuals to generate one-month market and firm i cumulative returns.[20] I use posterior means of parameters to compute time series volatility and correlations and similarly to Brownlees and Engle (2017) find standardized innovations for the market and the firm:

$$\epsilon_{mt} = \frac{r_{mt} - \mu_m}{\sigma_{mt}}$$

$$e_{it} = \left(\frac{(r_{it} - \mu_i)}{\sigma_{it}} - \rho_{imt} \frac{(r_{mt} - \mu_m)}{\sigma_{mt}} \right) / \sqrt{1 - \rho_{imt}^2}$$

.

Bootstrapped standardized innovations with replacement are used for generating the future paths of returns. The paths for r_{it} and r_{mt} which result in simple cumulative return for the market below the threshold of $C = -10\%$ ($R_{m\ T+1:T+h} < C$) are used for computation of LRMES as Monte Carlo average of bank $R_{i\ T+1:T+h}$.

Table 10 shows SRISK results with and without simulation of LRMES. Column 2 is without simulation using previous approximation of LRMES while Column 3 uses simulations with bootstrapped residuals. The results are of the same order for the averages

[19] Additional results not presented in the paper indicate that the same pattern happens at other dates in periods of high volatility.

[20] Bootstrap is used in order to capture fat tails of returns.

but estimated standard deviations for simulated LRMES and SRISK values are more than 10 times higher. If simulated systemic risk measures were used BAC and JPM would be ranked in the same bucket while using approximation without simulation results in separate rankings for one month LRMES and SRISK. Moreover, due to large standard deviations in simulated measures buckets would include more firms.

6.2 SRISK adjusted for illiquidity

Since the liquidity risk plays a major role during financial crises I extend SRISK measure to include the relative spread based on bid-ask closing prices: $S = \frac{ASK-BID}{Mid}$. In order to compute relative spread I use the Bid and Ask daily closing prices from CRSP and their average. The adjustment is similar to liquidity adjusted VaR (see e.g. Bangia et al. (1999)) subtracting $S/2$ from the returns. Figure 12 shows the relative spread for BAC and JPM, while the summary statistics are in Table 9. The relative spreads were the highest at the burst of the internet bubble declining to low levels by the end of 2003 and staying low till the 2007-2009 crisis resulting in multiple peaks. There is also some increase in relative spread for BAC around 2011 US Government shutdown. Because the spreads are positive and highly skewed to the right I show the median among other summary statistics in Table 9. The median for BAC is about 2.5 times higher than for JPM. Higher liquidity risk for BAC may potentially result in higher expected shortfall.

Figure 12 here

Table 10 shows results with and without illiquidity adjustment for one month LRMES and SRISK in columns 4 and 5. Results without simulations in column 4 are almost identical to results in column 2. This is due to approximation of LRMES that uses β rather than simulated cumulative returns from daily returns. On the other hand, as expected when simulation is used for illiquidity adjusted returns in column 5 the LRMES and SRISK average results are always higher than in column 3 without illiquidity adjustment. However, since the standard deviations are of the same order as before with simulations BAC and JPM are in the same bucket. Overall, the illiquidity adjustment is important to consider even if LRMES and SRISK results are not showing statistically significant difference. As it is a common practice to adjust capital requirements with illiquidity measures different methods of computing market and funding liquidity may be considered (such as Amihud (2002) and Brunnermeier and Pedersen (2009) among others). This would be an interesting topic for future research.

Tables 9 and 10 here

7 Conclusion

In this paper I introduced Bayesian analysis of systemic risk measures, derived posterior distributions and showed how to distinguish risks of different financial institutions. An asymmetric GTARCH model that generalizes popular GJR-GARCH model was estimated for equities and CDS spreads.

I find that systemic risks measures distributions could be highly overlapping, especially, during periods of higher volatility. Financial institutions can be grouped by buckets of non-overlapping posterior distributions of systemic risk measures for the assessment of the severity they impose on the financial system. In the future research it would be interesting to compare capital buckets set by regulators with clusters of firms based on systemic risk distributions that overlap.

More work needs to be done on various systemic risk measures accounting for uncertainty of estimated parameters using methods such as introduced in this paper. LRMES and SRISK measures after accounting for uncertainty of parameters can't distinguish rankings and capital requirements for the largest financial institutions in several settings. Different illiquidity measures could be explored and more international financial firms can be analyzed. It would be also interesting to consider different distributional assumptions for the error term.

Appendix. Markov Chain Monte Carlo Algorithms

First, I present MCMC algorithm for the GTARCH model in equation (4). For each firm and the market I estimate parameters in blocks: μ, $\alpha = (\omega, \alpha_1, \alpha_2)$ and $\beta = (\beta_1, \beta_2)$ using random walk draws. Let $\eta = (\mu, \alpha, \beta) = (\mu, \omega, \alpha_1, \alpha_2, \beta_1, \beta_2)$ the vector of all GTARCH parameters from equation (4).

The prior probability for the GTARCH volatility model be given by

$$\pi(\eta) \quad \propto \quad \mathrm{N}(\mu_0, \Sigma_\mu)\ \mathrm{N}(\alpha_0, \Sigma_\alpha)\ \mathrm{N}(\beta_0, \Sigma_\beta)$$

where μ_0, α_0, β_0 and $\Sigma_\mu, \Sigma_\alpha, \Sigma_\beta$ are hyperparameters of the mean and variance of the proper normal prior distribution of μ, α, β correspondingly. In practice we set the mean parameters equal to zero and large variances to make sure that the posterior distribution is similar to likelihood.[21] The posterior pdf is proportional to the product of the prior and the likelihood:

$$p(\eta|data) \propto \pi(\eta) \times L_v(data|\eta)$$

where

$$log(L_v) = -0.5 \sum \left(nlog(2\pi) + log(\sigma_{i,t}^2) + \frac{r_{i,t}^2}{\sigma_{i,t}^2} \right)$$

While I assume Normal distribution for the likelihood following Brownlees and Engle (2017), for the large sample n results for the GARCH parameters would be similar with other distributional assumptions.[22]

In the second step I perform MCMC estimation for the parameters of the correlation model in equation (9): $\psi = (\alpha_C, \beta_C)$. The prior pdf $\pi(\psi)$ is a similar proper Normal with zero means and large variances for hyperparameters. The posterior pdf of the DCC model with fitted GTARCH volatility is

$$p(\psi|data) \propto \pi(\psi) \times L_c(z_{i,t}, z_{m,t}|\psi)$$

where the correlation log likelihood is given by

$$log(L_c) = -0.5 \sum \left(log(1 - \rho_{i,t}^2) + \frac{z_{i,t}^2 + z_{m,t}^2 - 2\rho_{12,t} z_{i,t}^2 z_{m,t}^2}{1 - \rho_{i,t}^2} \right)$$

Overall, MCMC estimation takes two steps.

Step 1: I estimate parameters in blocks: μ, $\alpha = (\omega, \alpha_1, \alpha_2)$ and $\beta = (\beta_1, \beta_2)$ for each asset GTARCH model using random walk draws.

[21] I tried variances ranging from $10 - 10^6$ and convergence to the same posterior distributions of parameters was achieved.

[22] Using analogue of the quasi-maximum likelihood approach.

Step 2: using fitted volatilities from step 1 find standardized returns z_{it}, z_{mt} and estimate dynamic correlation between firm i and the market m. I estimate parameters in blocks using random walk draw: (i) ARCH parameters: α_{Ci} , (ii) GARCH parameters β_{Ci}.[23]

Each step is a separate MCMC chain and careful tests of convergence are applied.[24] Using posterior means and modes of parameters I also estimate the Modified Bayesian Information Criterion (MBIC).[25] I choose the best model by minimizing MBIC. This criterion is a Bayesian analogue of Akaike information criterion with ν degrees of freedom.

The modified Bayesian information criterion is given by:

$$MBIC = -2 \ln\, m(x) + 2(\nu + 1)$$

where the marginal likelihood $m(x)$ is computed by the Laplace-Metropolis estimation and evaluated at either posterior mean or mode.[26] The MBIC criterion is an in-sample information criterion showing overall fit of the models penalizing for additional parameters. Other model selection methods, such as posterior predictive density scores of Geweke and Keanne (2007) could be applied as well.

[23]There are two ways to estimate ω_{im}: as part of the ARCH block parameters or simply set it equal to the unconditional correlation of the firm and the market as is done in Brownlees and Engle (2017).

[24]I use the graphs of draws, fluctuation test (see Goldman and Tsurumi (2005)) and the acceptance rates to judge convergence.

[25]MBIC was introduced in Goldman and Tsurumi (2005).

[26]Alternatively one can use Chib and Jeliazkov (2001) estimator of marginal likelihood.

References

Acharya, V., Pedersen, L., Philippon, T., and Richardson, M. (2017). "Measuring Systemic Risk," *Review of Financial Studies*, 30 (1), 2-47.

Acharya, V., Engle, R. and Richardson, M. (2012). "Capital Shortfall: A New Approach to Ranking and Regulating Systemic Risks," *American Economic Review* 102 (3), 59-64.

Acharya, V., Engle, R., and Pierret, D. (2014). "Testing Macroprudential Stress Tests:The Risk of Regulatory Risk Weights," *Journal of Monetary Economics* 65, 36-53.

Adrian, T., and M. K. Brunnermeier. (2016) "CoVaR." *American Economic Review*, 106 (7), 1705-1741.

Allen, L., T. G. Bali, and, Y. Tang. (2012). "Does systemic risk in the financial sector predict future economic downturns?" *Review of Financial Studies*, 25, 3000 - 3036.

Amihud, Y. (2002). "Illiquidity and stock returns: cross-section and time-series effects," *Journal of Financial Markets,* 5(1), 31–56.

Arnold, B., Borio, C., Ellis, L., and F. Moshirian (2012). "Systemic risk, macroprudential policy frameworks, monitoring financial systems and the evolution of capital adequacy" *Journal of Banking & Finance,* 2012, 36 (12), 3125 - 3132.

Bangia, A., Diebold, F., Schuermann, T. and J. Stroughar (1999). "Liquidity on the outside," *Risk*, 12, 68-73.

Benoit, S., J.-E. Colliard, C. Hurlin, and C. Prignon (2017)." Where the risks lie: A survey on systemic risk." *Review of Finance*, 21 (1), 109-152.

Bisias, D., Flood, M., Lo, A. W., and Valavanis, S. (2012). "A survey of systemic risk analytics." Working paper #0001, Office of Financial Research.

Brownlees, C., and R. Engle (2017). "SRISK: A Conditional Capital Shortfall Measure of Systemic Risk." *Review of Financial Studies*, 30 (1), 48-79.

Brunnermeier, M. K. and Oehmke, M. (2013). "Bubbles, financial crises, and systemic risk." Handbook of the Economics of Finance. Elsevier.

Brunnermeier, M.K. and L.H. Pedersen (2009). "Market Liquidity and Funding Liquidity," *Review of Financial Studies,* 22 (6), 2201–2238.

Carr, P. and Wu, L. (2011). A Simple Robust Link Between American Puts and Credit Protection, *Review of Financial Studies,* 24(2), 473-505.

Chen, C. W.S. and So, M.K.P. (2006). "On a threshold heteroscedastic model." *International Journal of Forecasting,* 22, 73-89.

Chib, S. and E. Greenberg (1994). "Bayes inference in regression models with ARMA(p, q) errors", *Journal of Econometrics,* 64, 183206.

Chib, S. and E. Greenberg (1995). "Understanding the Metropolis-Hastings Algorithm," *The American Statistician,* 49, 327-335.

Chib, S., and I. Jeliazkov (2001) "Marginal Likelihood from the Metropolis-Hastings Output," *Journal of the American Statistical Association,* 96, 270–281.

Engle, R. (2002). "Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroscedasticity models," *Journal of Business & Economic Statistics*, 20, 339–50.

Engle, R. (2009). Anticipating correlations: A new paradigm for risk management. Princeton, NJ: Princeton University Press.

Girardi, G. and Ergun, A.T. (2013). "Systemic risk measurement: Multivariate GARCH estimation of CoVaR." *Journal of Banking & Finance*, 37, 3169-3180.

Glosten, L. R., Jagananthan, R., and Runkle, D. E. (1993). "On the relation between the expected value and the volatility of the nominal excess return on stocks." *The Journal of Finance,* 48, 1779-1801.

Geweke, J. and Keane, M. (2007). "Smoothly mixing regressions." *Journal of Econometrics,* 138, 252–291.

Goldman, E. and Tsurumi, H. (2005). "Bayesian Analysis of a Doubly Truncated ARMA-GARCH Model." *Studies in Nonlinear Dynamics and Econometrics,* 9 (2), article 5.

Goldman, E., and T. Agbeyegbe. (2007). "Estimation of Threshold Time Series Models using Efficient Jump MCMC." In Bayesian Statistics and its Applications, edited by S. K. Upadhyay, U. Singh, and Dipak Dey, 241-253. New Delhi: Anamaya Publishers.

Goldman, E., Nam, J., Tsurumi, H. and Wang, J. (2013) "Regimes and Long Memory in Realized Volatility." *Studies in Nonlinear Dynamics and Econometrics*, 17(5), 521-549.

Huang, X., H. Zhou, and H. Zhu (2011) "Systemic risk contributions." *Journal of Financial Services Research*, 42, 55-83.

Hull, J.C., Predescu, M., White, A., (2004). "The relationship between credit default swap spreads, bond yields, and credit rating announcements." *Journal of Banking and Finance,* 28, 2789-2811.

Nakatsuma, T., (2000). "Bayesian analysis of ARMA-GARCH models: A Markov chain sampling approach", *Journal of Econometrics,* 95, 57-69.

Oh, D.H. and Patton,A. J. (2017). "Time-Varying Systemic Risk: Evidence From a Dynamic Copula Model of CDS Spreads," *Journal of Business & Economic Statistics,* 1 - 47.

Tse, Y.K. and Tsui, A.K.C. (2002). "A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time- Varying Correlations," *Journal of Business & Economic Statistics,* 20 (3), 351 - 362.

Table 1: VLAB Systemic Risks for US institutions

Institution	SRISK%	RNK	SRISK ($ m)	LRMES	Beta	Cor	Vol	Lvg
			29-Aug-08					
Citigroup	12.89	1	132,039	73.64	2.61	0.79	63.4	19.99
JPMorgan Chase	9.37	2	96,045	70.95	2.42	0.74	62.9	13.42
Bank of America	9.24	3	94,637	77.27	2.9	0.74	75	11.94
Freddie Mac	6.74	4	69,069	92.26	5.01	0.44	221.2	297.76
American International Group	6.62	5	67,811	83.01	3.47	0.69	97	17.62
Merrill Lynch	6.6	6	67,588	82.66	3.43	0.78	83.8	22.45
Fannie Mae	6.56	7	67,156	94.01	5.51	0.51	205.4	115.68
Morgan Stanley	6.39	8	65,416	65.62	2.09	0.74	53.7	23.01
Goldman Sachs	5.63	9	57,676	58.04	1.7	0.75	43.3	16.99
Wachovia Bank	5.09	10	52,131	79.05	3.06	0.66	87.3	22.4
Lehman Brothers	4.71	11	48,249	92.18	4.99	0.74	130.2	55.88
MetLife	2.4	12	24,589	51.59	1.42	0.79	34.4	14.56
Prudential Financial	2.12	13	21,714	49.82	1.35	0.72	36.1	15.39
Washington Mutual	2.03	14	20,787	74.95	2.71	0.45	119.8	41.5
			31-Mar-09					
Bank of America	17.16	1	160,739	85.42	3.77	0.74	195.9	48.7
Citigroup Inc	14.12	2	132,262	85.42	3.77	0.66	219.3	121.21
JPMorgan Chase	13.91	3	130,281	75.58	2.76	0.8	133.1	20.11
Wells Fargo	8.94	4	83,752	80.99	3.25	0.73	170.5	20.53
American International Group	6.21	5	58,141	75.83	2.78	0.44	252.2	55.88
Goldman Sachs	5.47	6	51,257	60.93	1.84	0.8	88.3	18.58
Morgan Stanley	4.28	7	40,046	73.09	2.57	0.77	127.6	24.41
MetLife	3.64	8	34,045	80.2	3.17	0.73	168.5	26.12
Prudential Financial	3.45	9	32,280	88.65	4.26	0.74	221	52.34
Hartford Financial	2.25	10	21,095	84.34	3.63	0.72	194.2	106.08
			31-Dec-12					
Bank of America	17.98	1	100,700	53.52	1.5	0.66	28.8	16.4
Citigroup Inc	15.03	2	84,188	48.26	1.29	0.66	24.9	16.02
JPMorgan Chase	14.81	3	82,949	43.57	1.12	0.75	18.8	13.69
MetLife	8.62	4	48,306	56.95	1.65	0.73	28.6	22.75
Goldman Sachs	7.62	5	42,680	52.08	1.44	0.74	24.6	15.14
Prudential Financial	7.05	6	39,517	51.59	1.42	0.75	23.2	26.44
Morgan Stanley	6.93	7	38,838	51.62	1.42	0.69	25.9	19.42
Hartford Financial	3.34	8	18,721	54.23	1.53	0.73	26.3	30.17
American International Group	2.34	9	13,109	52.56	1.46	0.62	29.4	9.6
Lincoln National	2.31	10	12,925	52.81	1.47	0.75	24.7	29.11

Source: http://vlab.stern.nyu.edu

Table 2: Summary statistics for daily equity and CDS log returns

	BAC	CIT	JPM	SPX	CDS BAC	CDS JPM
mean	0.045	-0.009	0.047	0.011	0.053	0.028
std	3.406	3.673	2.841	1.342	5.108	4.203
Skew	0.904	1.468	0.829	0.017	-0.218	-0.175
Kurt	26.08	42.668	15.931	11.143	14.097	16.542
AR(1)	-0.011	0.046	-0.089	-0.091	-0.024	0.052

Notes: Returns are measured in basis points. Equity returns data are for the period 1/04/2001-12/31/2012 from CRSP database. CDS data are for the same period from Bloomberg.

Table 3: S&P500 volatility models results

	GTARCH		GJR-GARCH		GTARCH0	GARCH
	constrained	unconstrained	constrained	unconstrained		
μ	-0.018 (0.013)	-0.022 (0.013)	-0.009 (0.016)	-0.016 (0.016)	0.001 (0.015)	0.044 (0.016)
ω	0.031 (0.003)	0.029 (0.003)	0.030 (0.004)	0.027 (0.003)	0.036 (0.004)	0.035 (0.004)
α_1	0.161 (0.014)	0.158 (0.018)	0.189 (0.018)	0.188 (0.017)	0.087 (0.009)	0.113 (0.009)
α_2	0.005 (0.004)	-0.011 (0.008)	0.003 (0.003)	-0.021 (0.007)	0.087 (0.009)	0.113 (0.009)
β_1	0.975 (0.017)	0.977 (0.018)	0.886 (0.009)	0.901 (0.010)	1.022 (0.018)	0.863 (0.010)
β_2	0.834 (0.012)	0.854 (0.015)	0.886 (0.009)	0.901 (0.010)	0.772 (0.015)	0.863 (0.010)
$(\alpha_1 + \alpha_2 + \beta_1 + \beta_2)/2$	0.987 (0.004)	0.989 (0.004)	0.982 (0.005)	0.985 (0.004)	0.984 (0.005)	0.976 (0.005)
volf= $\sqrt{252\sigma^2_{T+1}}$	15.121 (0.382)		13.857 (0.260)		16.098 (0.474)	13.243 (0.207)
95% HPDI for volf	[14.384–15.856]		[13.347–14.371]		[15.126–17.056]	[12.831–13.642]
Correl $(r_{t-1}, log(\sigma^2_t/\sigma^2_{t-1}))$	-0.738		-0.650		-0.508	-0.116
MBIC at mean	**3274.45**	3268.55	3412.85	3284.85	3357.71	3431.54
MBIC at mode	**3224.72**	3224.25	3375.40	3242.54	3318.52	3397.04

Notes: Data for the S&P500 index for the period 01/04/2001-12/31/2012. All coefficients are reported at posterior means and standard deviations are given in brackets. I derive posterior distributions of 1 day out of sample volatility forecast ($\sqrt{252\sigma^2_{T+1}}$) using MCMC draws of parameters. MBIC is the Modified Bayesian Information Criterion.

Table 4: CDS JPM volatility models results

	GTARCH	GJR-GARCH	GTARCH0	GARCH
μ	-0.054 (0.038)	-0.074 (0.057)	-0.054 (0.036)	-0.095 (0.056)
ω	0.401 (0.059)	0.374 (0.059)	0.410 (0.056)	0.370 (0.050)
α_1	0.114 (0.015)	0.104 (0.014)	0.127 (0.009)	0.125 (0.009)
α_2	0.132 (0.012)	0.137 (0.014)	0.127 (0.009)	0.125 (0.009)
β_1	0.838 (0.017)	0.872 (0.010)	0.828 (0.015)	0.870 (0.008)
β_2	0.900 (0.013)	0.872 (0.010)	0.903 (0.012)	0.870 (0.008)
$(\alpha_1 + \alpha_2 + \beta_1 + \beta_2)/2$	0.991 (0.005)	0.993 (0.004)	0.992 (0.005)	0.995 (0.004)
volf= $\sqrt{252\sigma^2_{T+1}}$	51.440 (1.219)	50.681 (1.053)	50.681 (1.053)	51.148 (0.607)
95% HPDI for volf	[48.925–53.755]	[48.300–52.434]	[48.300–52.434]	[49.974–52.341]
Correl $(r_{t-1}, log(\sigma^2_t/\sigma^2_{t-1}))$	0.177	0.147	0.142	0.061
MBIC at mean	11358	11358	**11351**	11354
MBIC at mode	11318	11333	**11317**	11326

Notes: Data for JPM CDS for the period 09/06/2001-12/31/2012. All coefficients are reported at posterior means and standard deviations are given in brackets. I derive posterior distributions of 1 day out of sample volatility forecast ($\sqrt{252\sigma^2_{T+1}}$) using MCMC draws of parameters. MBIC is the Modified Bayesian Information Criterion.

Table 5: Estimation results for DCC-GJR-GARCH model

	BAC	CIT	JPM
α_C	0.130 (0.013)	0.063 (0.012)	0.055 (0.017)
β_C	0.798 (0.027)	0.873 (0.033)	0.747 (0.134)
Correlation forecast	0.610 (0.011)	0.645 (0.018)	0.750 (0.012)
Beta forecast	1.285 (0.043)	1.073 (0.043)	1.076 (0.032)
MES	0.038 (0.0008)	0.031 (0.0007)	0.028 (0.0005)
LRMES	0.481 (0.011)	0.422 (0.013)	0.423 (0.010)
SRISK $\times 10^{10}$	9.810 (0.129)	7.220 (0.135)	8.366 (0.147)

Notes: Data for BAC, CIT, JPM and S&P500 index for the period 01/04/2001-12/31/2012. All coefficients, forecasts of correlation, beta, MES, LRMES and SRISK are reported at posterior means and standard deviations are given in brackets.

Table 6: JPM Risks for various volatility models

	GTARCH	GJR-GARCH	GTARCH0	GARCH
Beta	0.997 (0.036)	1.076 (0.032)	1.002 (0.043)	1.152 (0.044)
MES	0.028 (0.0006)	0.028 (0.0005)	0.030 (0.0007)	0.029 (0.0005)
LRMES	0.399 (0.011)	0.423 (0.010)	0.401 (0.013)	0.445 (0.012)
SRISK $\times 10^{10}$	8.000 (0.168)	8.366 (0.147)	8.023 (0.203)	8.700 (0.190)
95% HPDI for SRISK	[7.654–8.344]	[8.081–8.661]	[7.612–8.429]	[8.314–9.080]

Notes: Data for JPM and SPX index for the period 01/04/2001-12/31/2012. All coefficients are reported at posterior means and standard deviations are given in brackets.

Table 7: 95% Highest Posterior Density Intervals (HPDIs) for GJR-GARCH Model

	Volatility	Beta	MES	LRMES	SRISK $\times 10^{10}$
			CASE 1: Low Volatility 2012/12/31		
BAC	[28.015–30.408]	[1.206–1.369]	[0.037–0.040]	[0.460–0.503]	[9.570–10.067]
CIT	[22.130–24.067]	[0.991–1.157]	[0.030–0.032]	[0.398–0.447]	[6.961–7.484]
JPM	[19.199–20.567]	[1.014–1.142]	[0.027–0.029]	[0.404–0.442]	[8.081–8.661]
			CASE 2: Medium Volatility 2008/08/29		
BAC	[48.075–55.482]	[2.211– 2.658]	[0.058–0.067]	[0.678–0.744]	[8.229–9.090]
CIT	[45.652–51.194]	[1.959– 2.402]	[0.056–0.063]	[0.634–0.708]	[12.229–12.935]
JPM	[49.562–54.189]	[2.013–2.374]	[0.060–0.066]	[0.643–0.703]	[8.797–9.530]
			CASE 3: High Volatility in 2009/03/31		
BAC	[183.072–205.059]	[3.716–4.628]	[0.221–0.248]	[0.852 –0.907]	[16.066–16.287]
CIT	[167.778–180.527]	[2.976–3.755]	[0.205–0.221]	[0.783–0.855]	[13.135–13.227]
JPM	[136.537–146.662]	[2.367–3.487]	[0.164–0.180]	[0.707–0.836]	[12.583–13.769]

Table 8: 6 Months SRISK and Components

	Quasi Leverage	Equity (Bill $)	Quasi Assets (Bill $)	Beta	SRISK (Bill $)
			CASE 1: Low Volatility 2012/12/31		
BAC	**17**	125	2096	**1.29** (0.04)	**98.10** (1.29)
CIT	15	116	1790	1.07 (0.04)	72.20 (1.35)
JPM	14	167	**2320**	1.08 (0.03)	83.66 (1.47)
			CASE 2: Medium Volatility 2008/08/29		
BAC	12	142	1695	**2.43** (0.11)	86.58 (2.20)
CIT	**20**	103	**2059**	2.19 (0.11)	**125.73** (1.75)
JPM	13	132	1771	2.18 (0.09)	91.58 (1.90)
			CASE 3: High Volatility 2009/03/31		
BAC	49	44	**2143**	**4.16** (0.23)	**161.77 (0.57)**
CIT	**121**	14	1697	3.37 (0.20)	131.82 (0.24)
JPM	20	100	2011	2.89 (0.28)	131.53 (2.93)

Table 9: Summary Statistics for Relative Spread based on bid-ask closing prices

	BAC	JPM
Mean	0.047	0.054
Median	0.019	0.008
Maximum	0.938	0.660
Minimum	0.000	0.000
Std. Dev.	0.086	0.110
Skewness	4.081	3.028
Kurtosis	22.808	12.143

Table 10: Comparison of 1 month SRISK with and without simulation and illiquidity adjustment

	returns		illiquidity adjusted returns	
	w/o simulation	with simulation	w/o simulation	with simulation
	posterior mean (stdev)	mean (stdev)	posterior mean (stdev)	mean (stdev)
	CASE 1: Low VOL 2012/12/31			
JPM				
LRMES	0.107 (0.003)	0.155 (0.093)	0.107 (0.003)	0.163 (0.094)
SRISK ($\times 10^{10}$)	3.512 (0.047)	4.243 (1.424)	3.512 (0.047)	4.369 (1.450)
BAC				
LRMES	0.127 (0.004)	0.172 (0.106)	0.127 (0.004)	0.178 (0.110)
SRISK ($\times 10^{10}$)	5.729 (0.047)	6.250 (1.224)	5.734 (0.046)	6.323 (1.262)
	CASE 2: Medium VOL 2008/08/29			
JPM				
LRMES	0.205 (0.008)	0.212 (0.124)	0.205 (0.008)	0.226 (0.127)
SRISK ($\times 10^{10}$)	3.463 (0.096)	3.545 (1.512)	3.463 (0.094)	3.715 (1.549)
BAC				
LRMES	0.226 (0.009)	0.189 (0.132)	0.226 (0.009)	0.200 (0.134)
SRISK ($\times 10^{10}$)	2.326 (0.120)	1.887 (1.669)	2.325 (0.121)	2.015 (1.712)

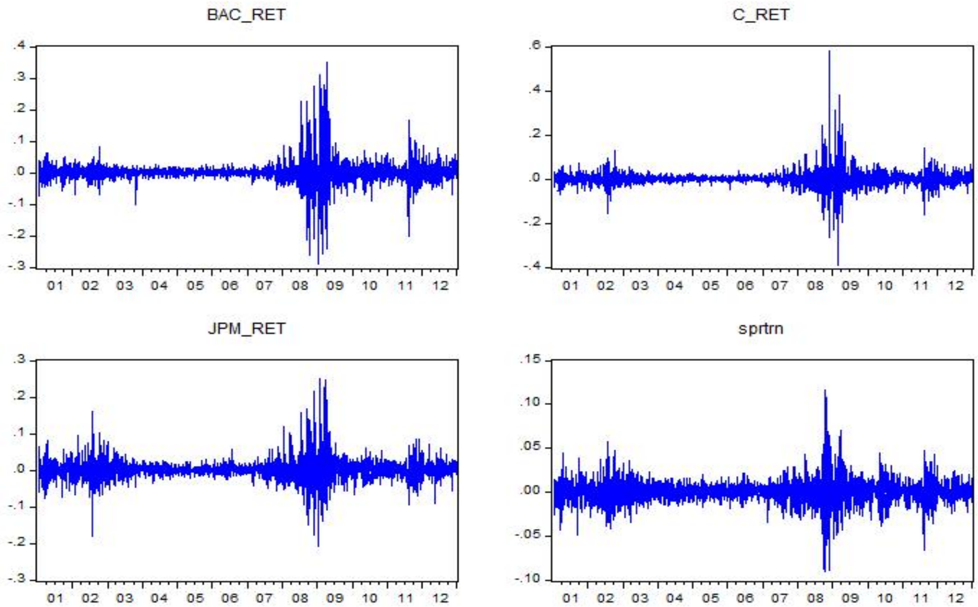

Figure 1: Returns: BAC, CIT, JPM, SPX

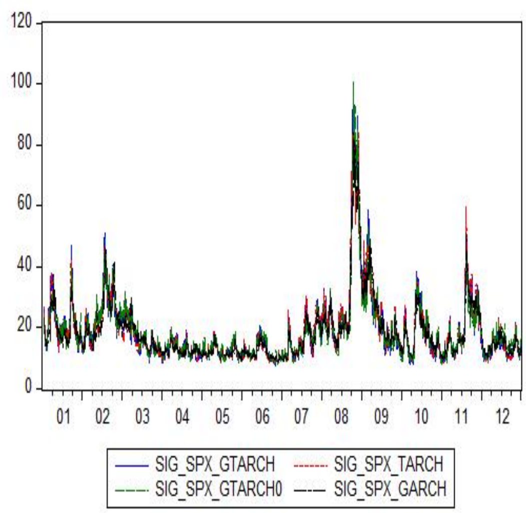

Figure 2: Annualized SPX volatility using all models

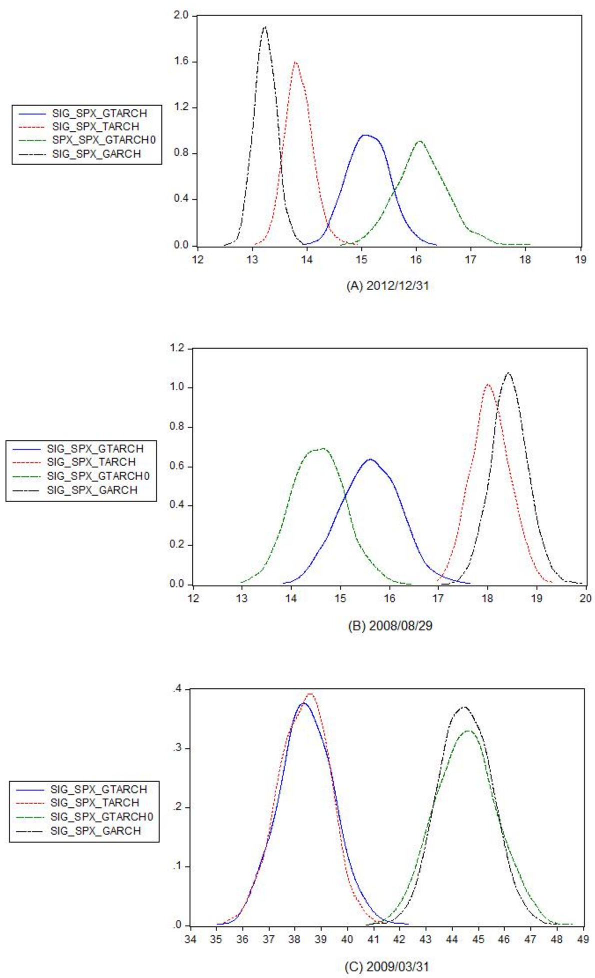

Figure 3: PDFs of one day forecasts of SPX volatility using all models: (A) 2012/12/31, (B) 2008/08/29, (C) 2009/03/31

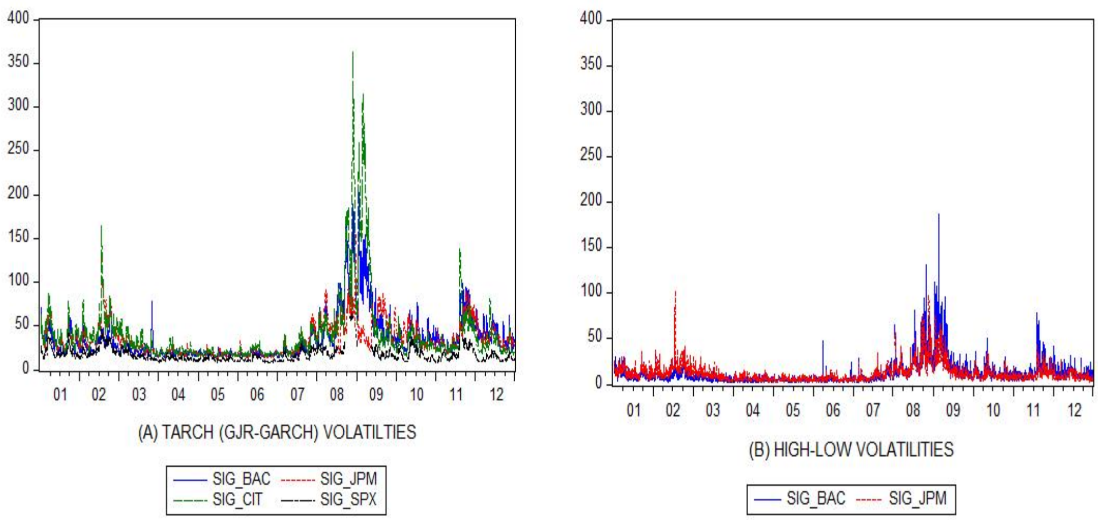

Figure 4: (A) GJR-GARCH Volatility: BAC,CIT, JPM, SPX, (B) HIGH-LOW Volatility: BAC, JPM

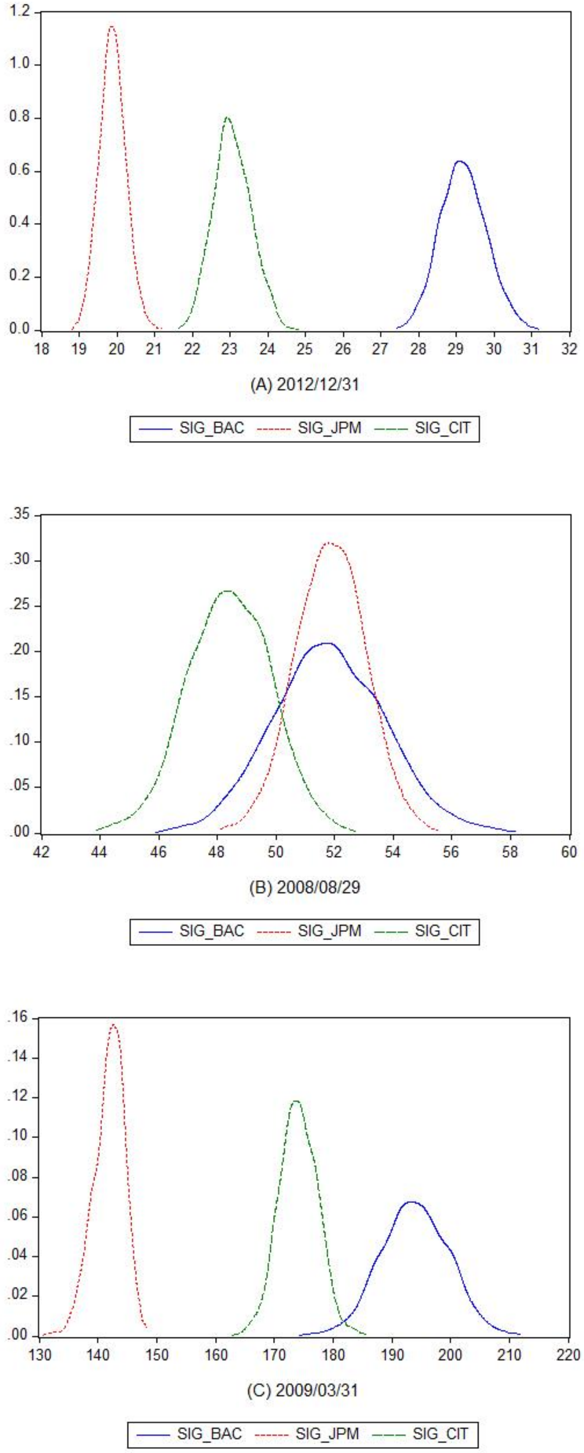

Figure 5: PDFs of one day forecasts of volatility for BAC, CIT and JPM using DCC-GJR-GARCH model: (A) 2012/12/31, (B) 2008/08/29, (C) 2009/03/31

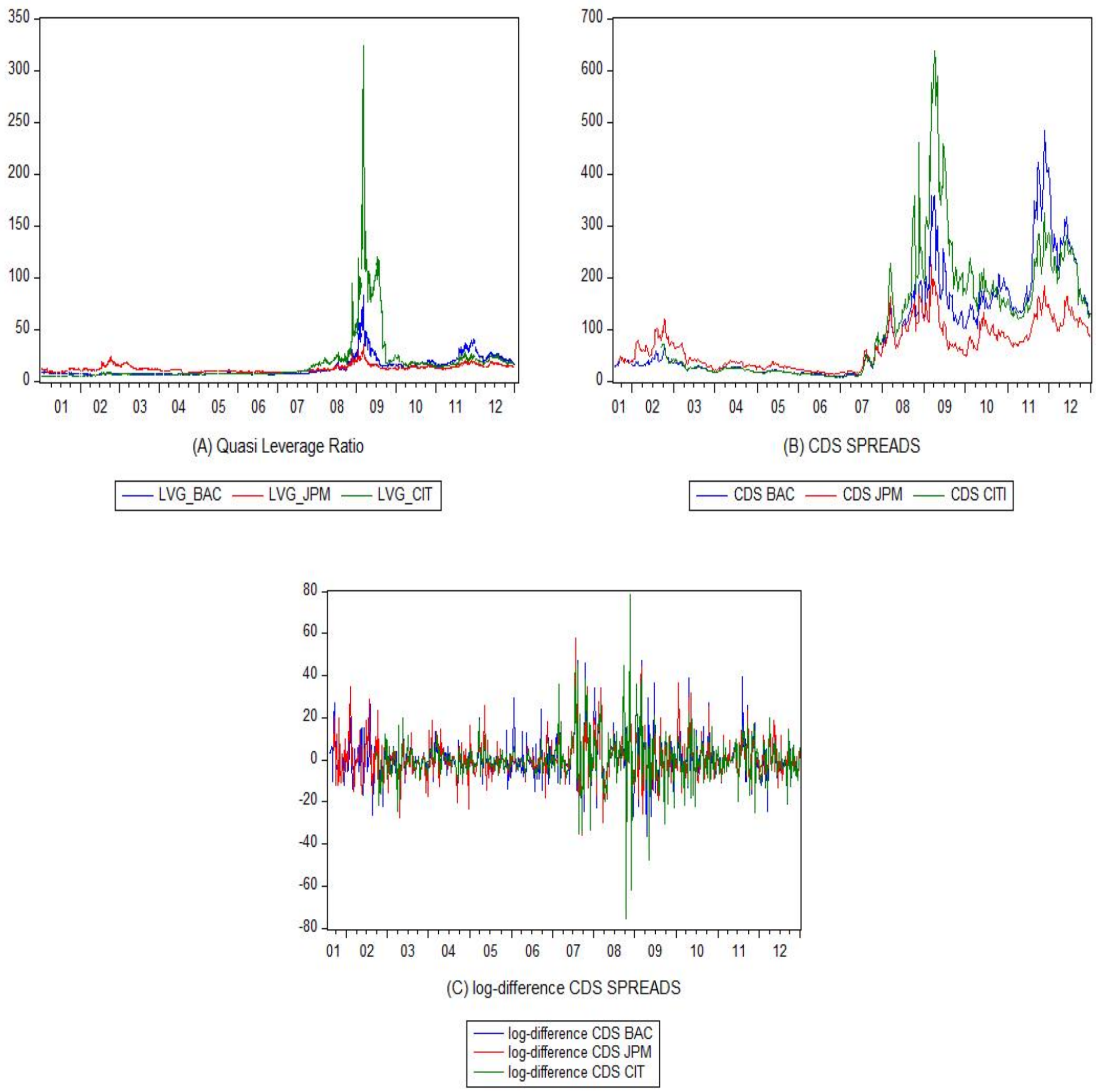

Figure 6: (A) Quasi Leverage Ratio $LVG = (D_t + E_t)/E_t$, (B) CDS spreads, (C) log-difference of CDS spreads

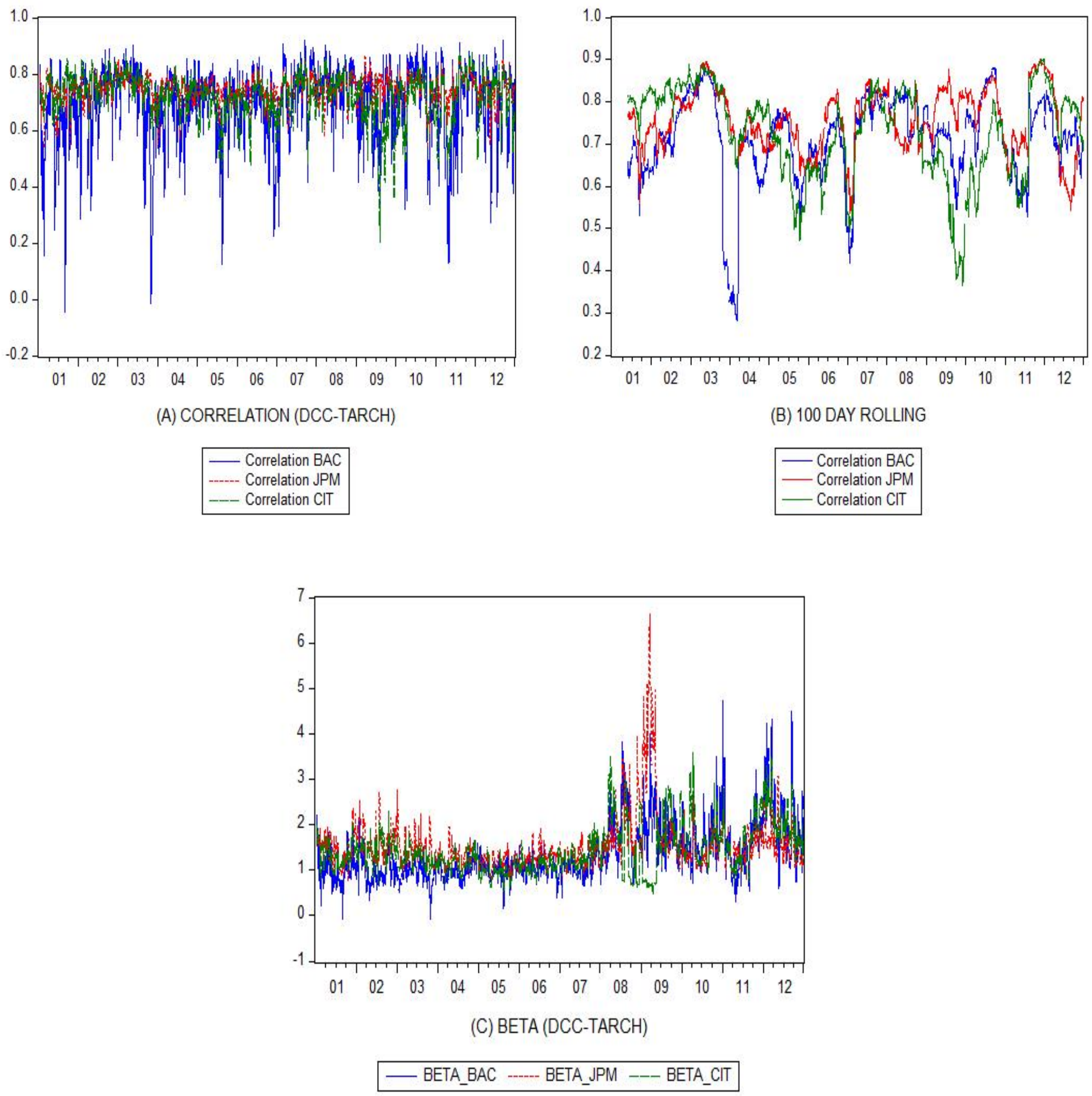

Figure 7: (A) Dynamic correlation with the market (DCC-GJR-GARCH), (B) 100 day rolling correlation with the market, (C) Dynamic estimates of beta (DCC-GJR-GARCH)

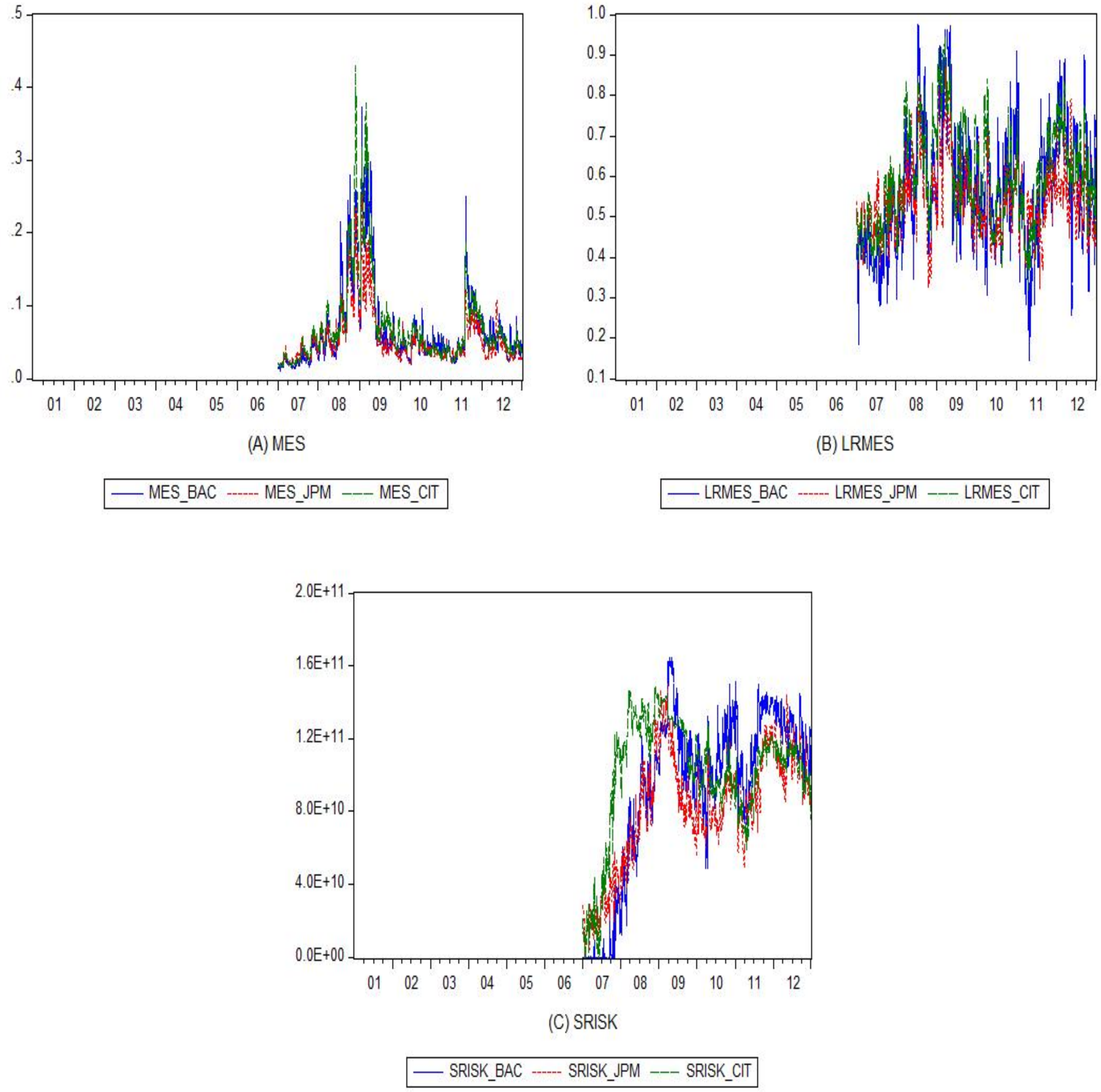

Figure 8: MES, LRMES and SRISK based on GJR-GARCH model: BAC,CIT, JPM

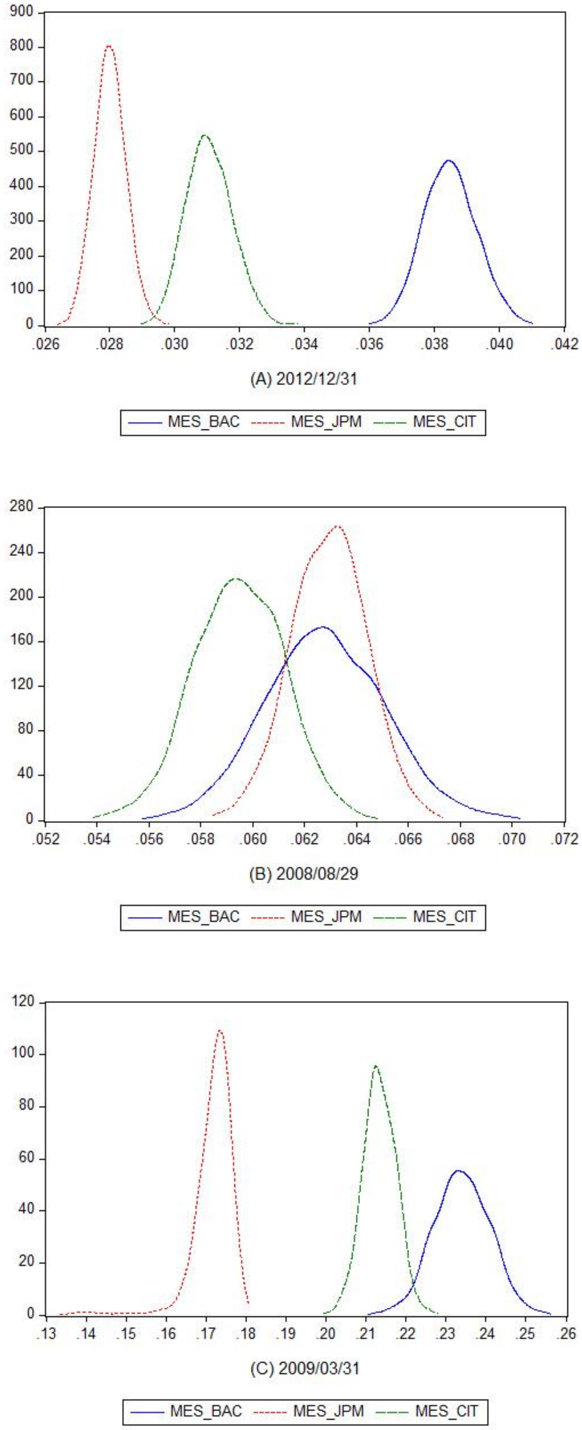

Figure 9: PDFs of Marginal Expected Shortfall for BAC, CIT and JPM using DCC-GJR-GARCH model: (A) 2012/12/31, (B) 2008/08/29, (C) 2009/03/31

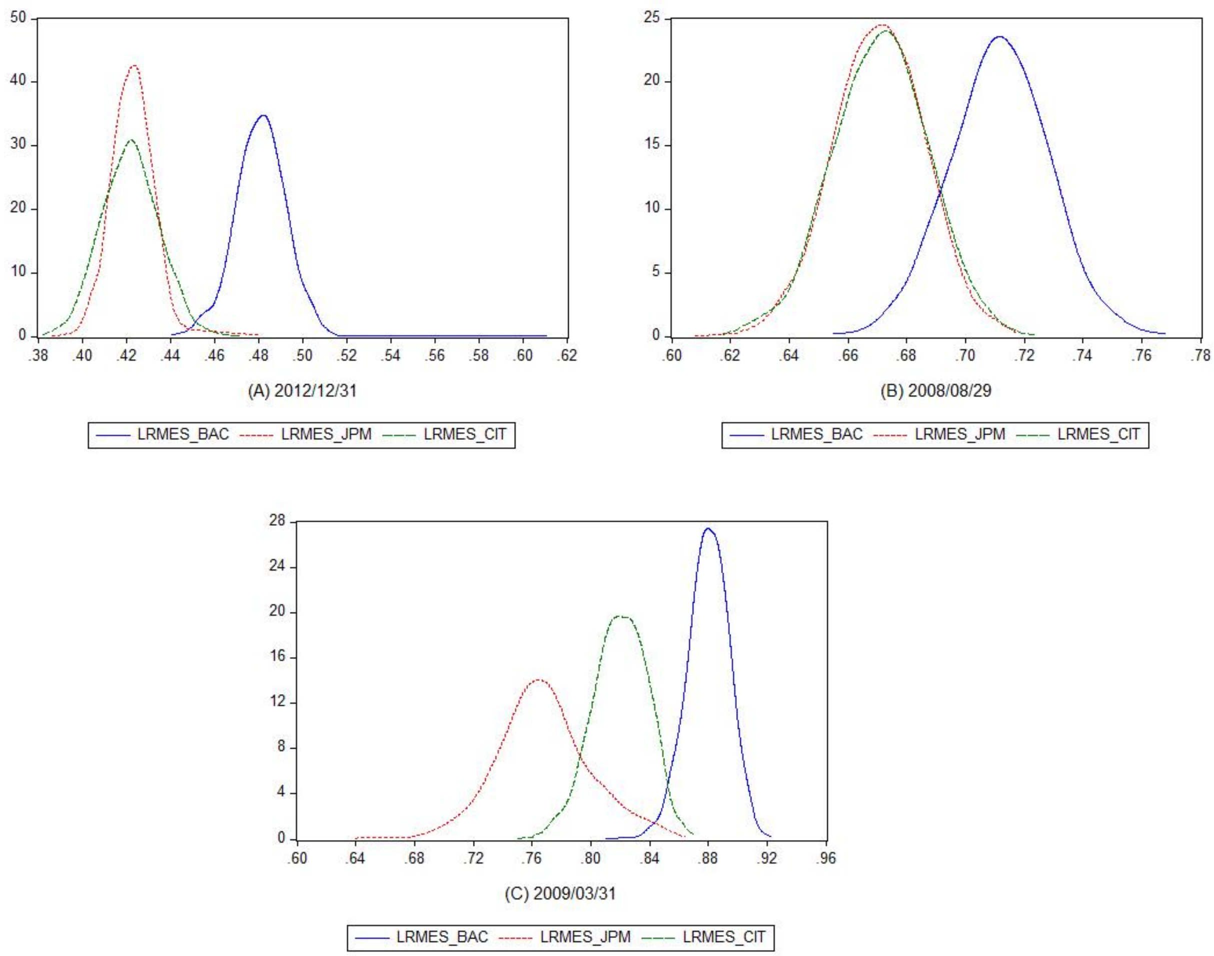

Figure 10: PDFs of Long Run Marginal Expected Shortfall for BAC, CIT and JPM using DCC-GJR-GARCH model: (A) 2012/12/31, (B) 2008/08/29, (C) 2009/03/31

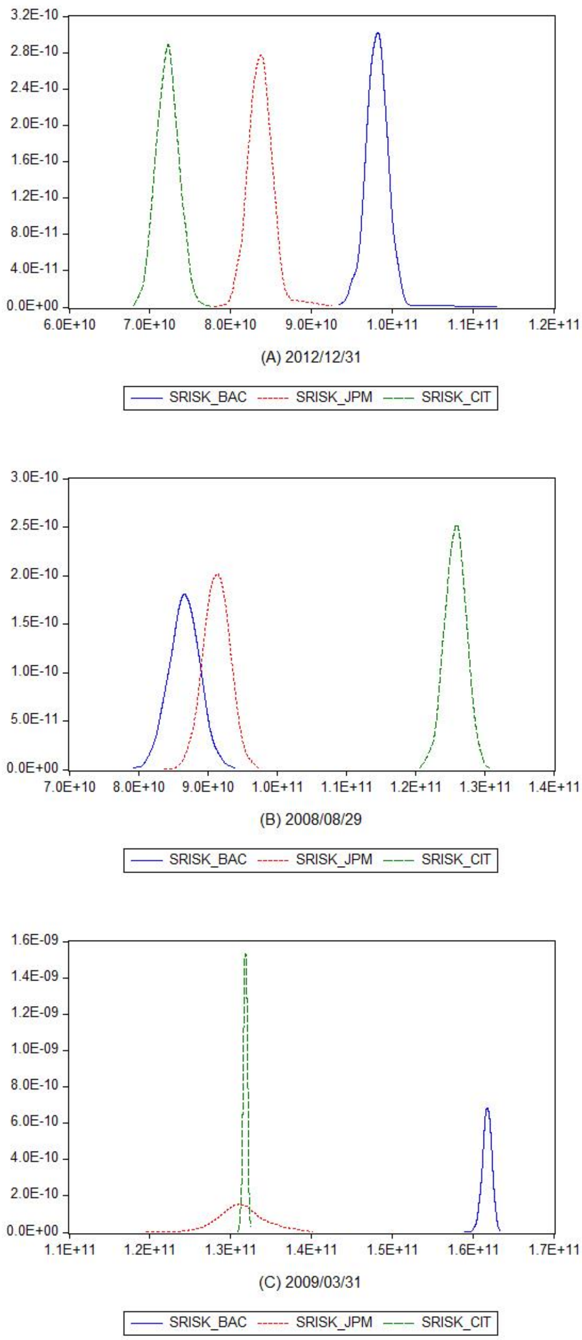

Figure 11: PDFs of SRISK for BAC, CIT and JPM using DCC-GJR-GARCH model: (A) 2012/12/31, (B) 2008/08/29, (C) 2009/03/31

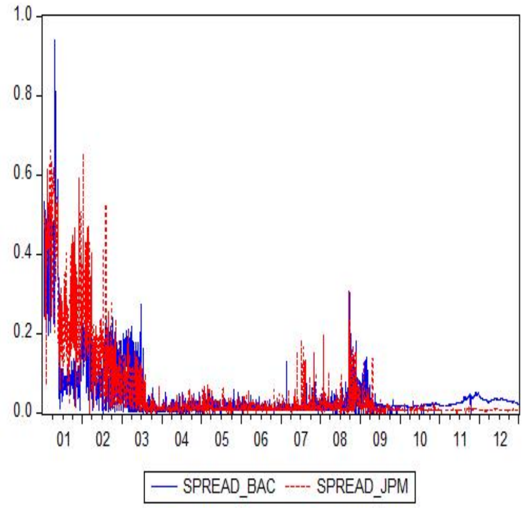

Figure 12: Relative Spread: BAC, JPM