AGENCY PROBLEMS OF DEBT, CONVERTIBLE SECURITIES, AND
DEVIATIONS FROM ABSOLUTE PRIORITY IN BANKRUPTCY

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ABSTRACT

It has been suggested that convertible debt can be used to reduce the tendency towards excessive risk-taking in a firm that includes debt in its capital structure. We show that the ability of convertible debt to perform this function is greatly reduced if stakeholders can trade in derivative securities with payoffs contingent on the cashflows of the firm. We show, further, that bankruptcy courts can ensure the same result precisely by deviating from absolute priority. Our model explains two real-world phenomena; one, why bankruptcy law seems to be structured in such a way as to favor equity holders and facilitate deviations from absolute priority, and two, why small firms are more likely to use convertible debt.
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I. INTRODUCTION

Following Jensen and Meckling's seminal article identifying the various agency problems which might arise as agents attempted to exercise their claims to the returns of a firm, many researchers have investigated the incentive for levered firms to substitute riskier projects for less risky ones. This incentive derives from the fact that shareholders' limited liability transforms levered equity into an option on the assets of the firm. As a result, when debt is issued, equityholders find it in their interest to choose assets whose return distributions have higher variances, even if the means are lower. However, by issuing a modified debt security, such as convertible debt, which gives bondholders a countervailing option, the incentives of shareholders to switch to riskier assets can be reduced. Such a strategy has been suggested by Jensen and Meckling themselves, and formalized later, by Green.

Our paper makes two points in this regard. First, we show that such solutions to this agency problem are undermined in a market environment in which individual investors as well as firms can trade in contingent claims. Second, even though market participants by themselves may not be able to eliminate these agency costs, we show that bankruptcy courts can mitigate these costs by systematically deviating from the absolute priority rule.
A. Agency Problems and Convertible Debt

Attempts by firm managers to use convertible debt to resolve debt-related agency problems are handicapped when the firm operates in a market environment where individuals can trade in contingent claims, and can "undo" the convertibility feature of the debt issued by the firm if they so desire. This point has not been recognized in the literature. The point, of course, is not that convertible debt can never help to resolve the agency problem. Rather, recognition of the circumstances under which it can or cannot resolve the problems helps us to understand cross-sectional variations in the use of convertible securities, in particular. For example, the use of convertible debt would be expected to be greatest for firms with a high level of insider ownership, since equity holders in such firms would be less likely to act independently of their fellow share holders. This may also explain the greater tendency for smaller firms to issue convertible debt since small firms are frequently closely owned. This would certainly be a preferred alternative to the currently ruling "sweetener" theory cited as an explanation for the above phenomenon.

B. Agency Problems and the Courts

While it may be that market participants by themselves cannot avoid the agency problems and hence the associated deadweight costs identified by Jensen and Meckling, it remains possible for an agent outside the system to impose the optimal solution on the market. It has been argued by many economists that one of the functions of a legal system is to induce economically efficient behavior whenever markets fail to achieve such a solution, for one reason or another. In Posner's words, "the law ....induce(s) people to efficiency." Similarly, Hirshleifer suggests that in a social equilibrium, the
law will choose the most efficient of the possible assignments of property rights, wherever unavoidable transactions costs preclude achievement of a fully efficient result by private negotiation.

In our model, we obtain the optimal risk sharing contract in an environment in which all investors have access to contingent claims markets. This contract is characterized by the bankruptcy court's imposition of an allocation of the firm's assets in bankruptcy states, in which equity holders receive a positive share. In non-bankruptcy states, the market solution prevails—that is, investors hold straight debt and equity. Under this contract, equity holders find they have no incentive to switch to riskier assets. The optimal contract derived in this paper then, is precisely what would result if bankruptcy courts were to deviate from absolute priority rules to award equity holders a share in the firm's assets.

We show that a system of legal procedures which favors equity holders in times of financial distress by providing them with a bargaining advantage, ensures that risk sharing in bad states will occur and as a consequence mitigates the agency costs of debt. Bankruptcy courts, in this way, are seen to represent a natural solution to the moral hazard problems associated with debt.

There is now substantial evidence that bankruptcy courts do not follow absolute priority rules. Franks and Torous, Weiss and Eberhart, Moore and Roenfeldt document that many firms in bankruptcy violate absolute priority rules. Furthermore, in a large proportion of these cases, the violations favor equity holders. For example, Eberhart, Moore and Roenfeldt document that, in their sample, the amount paid to shareholders in excess of that which they would have received under the absolute priority rule averages 7.6% of the total award to all claimants. Our model shows that such seemingly
irresponsible behavior by the courts in fact could very well increase economic efficiency by reducing agency costs.

Our model could also explain why the Bankruptcy Reform Act of 1978, which favors equity holders in bankruptcy, came into existence when it did. On the one hand, the economy had been growing more uncertain in the post oil-shock era implying greater the opportunities for excessive risk-seeking behavior. On the other hand, there had recently evolved a vast array of derivative securities such as stock options, and financial futures: the CBOE opened its doors in 1973, although stock options were traded prior to that over the counter, and financial futures trading began in 1975. The existence of these contingent claims suggests that the costs of creating new derivative securities were a lot lower than they had previously been. Consequently, the ability of investors to undo firms' financial decisions had increased, providing a need for an extra-market resolution to the agency problems of debt. Stock market uncertainty and the use of financial derivative securities have only increased since that time, implying that the use of convertible debt and other theoretically incentive compatible contracts should have decreased over the last decade or two. In fact, Lehn and Poulsen17 found that the ratio of convertible debt to straight debt declined during the eighties.

In the next section, we present our formal model. Since our purpose is to investigate the treatment of debt in bankruptcy, we begin with sufficient assumptions to generate the issuance of debt. We then introduce moral hazard which results in agency costs of debt. We show that if there are no contingent claims markets in which shareholders can trade, the manager can reduce these agency costs by issuing modified debt securities, which involve greater risk sharing. In section III, we show that this is not a sustainable market equilibrium if contingent claims markets exist, thereby opening up a
role for the judicial system to enforce the optimal amount of risk sharing. The final section concludes.

II. THE MODEL

Before laying out the former model, we present an intuitive explanation of our arguments. We first assume that investors have utility functions that require the issuance of debt for optimal risk-sharing. We then show that this engenders an agency problem in the form of inappropriate incentives for risk-taking. However, if investors are denied access to contingent claim markets, this problem can be resolved by issuing convertible debt. This salutary effect of convertible debt is, of course, well known and does not require much explanation. However, our model now goes beyond the current models in analyzing the more realistic scenario where investors do have access to contingent claim markets.

While convertible debt may succeed in solving the agency problem, it does not provide individual investors with their most preferred payoff pattern. In the absence of agency problems, the firm's shareholders prefer to hold equity levered by straight debt, while bondholders prefer to hold straight debt. Now, if investors are permitted to trade in contingent claims, a debt holder will split the convertible debt issued by the firm into its straight debt and warrant components, sell off the warrant and thereby achieve his most preferred payoff pattern. Moreover, since by issuing convertible debt the firm has committed itself to choosing the less risky asset (when the opportunity arises), this investor can achieve what was previously not available to him—his first best consumption pattern; that is, his preferred consumption pattern in the absence of the agency problem. A similar analysis would apply to the equity holder as well. Hence, when the firm issues
convertible debt, each investor has the incentive to free ride by obtaining the benefits of
the resolution of the moral hazard problem without bearing the costs of the suboptimal
risk sharing that the issue of convertible debt implies.

The result of each investor trading on his own account, of course, is that all share
holders trade away from equity levered by the convertible debt contract and trade back to
equity levered by straight debt. Consequently, a manager who maximizes the wealth of
the firm's equity holders will have the same incentive to substitute into riskier projects
after convertible debt has been issued, as he had prior to the issuance of convertible debt.
Any attempt by the manager to reduce the agency problem by altering the securities
issued by the firm is thus compromised when all parties have access to contingent claims
markets.

Although market participants by themselves fail to resolve the agency problem, an
external agent in the form of the bankruptcy court, can impose a contract that resolves the
agency problem. We show that the optimal method by which the court can achieve this is
precisely to deviate from absolute priority. We now proceed to the formal model.

A. The Standard Debt Contract

To understand the effect that court decisions have on the securities market, we
have to consider that market in the absence of judicial intervention. Consider a three date
world in which there are two types of agents, who differ in both endowments as well as
preferences. For the moment we consider only agents' endowments, leaving a
description of their preferences for later. At time 0, each agent is endowed with a single
type of security. Type A agents own all the shares in a firm which consists of a single
asset whose random payoff, $X$, is realized at time 2. Type B agents are endowed with bonds exogenous to the firm, which provide a cash payment of $W$ with certainty at time 2. At time 0, agents are permitted to trade in the shares of the firm or in the exogenous bonds, as well as in any other securities that the manager of the firm may want to issue. At time 1, the manager, acting in the best interests of the shareholders, may have the option to exchange the firm’s asset for another asset whose (different) random payoff is also realized at time 2. We assume for now that agents cannot trade in contingent claims.

Since agents of each type are homogenous, we can assume without loss of generality, that there is only one agent of each type. We can also dispense with the manager temporarily—if he acts in agent A’s interest and there is only one agent A, the manager is for all purposes identical with agent A. We will also assume, for the moment, that the option to switch assets is unavailable. In such an environment, the two agents will enter into contracts to take advantage of their abilities to share risk in such a way as to Pareto-improve on their original endowments. These Pareto-optimal contracts will be composed of some payment $P$ from agent B to agent A in exchange for a portion, $y(X)$, of the asset’s payoff which solves the problem:

\[
\begin{align*}
\text{Max}_{y} & \int U_A(X - y + P)dF(X) \quad \text{(1a)} \\
\text{subject to agent B's Individual Rationality constraint,} & \\
\int U_B(W + y - P)dF(X) = u_B \quad \text{(1b)}
\end{align*}
\]

where $U_i(.)$, $i = A,B$ are the utility functions of A and B respectively, $u_B$ is agent B’s reservation utility level and $F(.)$ is the cumulative density of $X$ over its domain (the positive reals). It is assumed that agents’ utility functions have the usual derivative.
properties: $U' > 0$ and $U'' \leq 0$. The admissible class of functions $y(X)$, among which the maximum is sought, consists of all functions such that $0 \leq y(X) \leq X$, for all $X$.

The solution to the maximization problem in (1) is characterized below. The intuition is straightforward. Since the essence of the contract is risk-sharing, the optimal contract assigns shares in the asset so that the agent bearing the greatest risk is the one most willing to do so, that is, the less risk averse agent.

**Proposition 1:**

The optimal solution to the asset owner's problem (1) satisfies:

$$\frac{\partial y^*}{\partial X} = \frac{R_A}{R_A + R_B}$$  \hspace{1cm} (2)

where $R_i, i = A,B$, is the Arrow-Pratt measure of absolute risk aversion.

At this point, we assume a convenient set of preferences for the agents which permits the generation of the standard debt contract given by $\min(X,D)$. To this end, we assume that the agents' preferences can be described by the following functions:

$$U_A(.) = \begin{cases} \Psi_A(.) & \text{for } X \leq D \\ \Phi_A(.) & \text{for } X > D \end{cases}$$  \hspace{1cm} (5a)

$$U_B(.) = \begin{cases} \Phi_B(.) & \text{for } X \leq D \\ \Psi_B(.) & \text{for } X > D \end{cases}$$  \hspace{1cm} (5b)

where $\Psi_i$ and $\Phi_i, i = A,B$, are concave and linear utility functions respectively. In good states of the world ($X > D$) the asset owner, agent A, is risk neutral and agent B is risk averse, while in bad states of the world ($X \leq D$) the reverse is true.\footnote{We are now ready to characterize the optimal contract.}
Proposition 2: If the agents’ utility functions are given by (5), the standard debt contract is optimal with the face value of debt equal to $D$.

The intuition once again is clear. Since agent A is risk neutral in good states of the world ($X > D$), he bears all the risk, while agent B receives a fixed payment. In bad states of the world ($X < D$), agent B is risk neutral and hence he bears all the risk, while agent A receives a fixed payment. The shares received by each agent, as a function of the asset's return, are depicted in Figure 1.

Now, suppose that agent A has the opportunity to switch to another asset at time 1. Assume that the payoffs, $X_i$, to the two assets can be represented by normal distributions, $N(\mu_i, \sigma_i)$, $i = 1, 2$, with the same mean and different variances. Let the existing asset, denoted now as asset 1, have the lower variance. Then, choosing to remain with the existing asset would lead to a higher firm value if no moral hazard were present. However, we can show that if the switching option became available to agent A, he would choose the higher variance asset.

Proposition 3: Suppose the random payoffs of two assets are normally distributed with equal means and different variances. Then all risk averse individuals would prefer levered equity in the asset with the greater variance to that with the lower variance.
Since agent A's utility function is concave over part of its domain, it follows that he will switch to the higher variance asset. Agent B, however, is aware of the incentive agent A has to switch to the riskier asset and so would accept the standard debt contract only at a lower price \( P_2 \). At this price agent A will continue to select asset 2. Since asset 1 has greater value, this is inefficient and in neither agent A's interest nor in agent B's; both of them would prefer to create appropriate incentives so that agent A will, in fact, choose asset 1. It is the conflict of interest engendered by the standard debt contract which makes agent A prefer asset 2 to asset 1. An obvious way to remove this conflict of interests is to ensure that agent B's utility is independent of agent A's choice of asset. If the sharing rule is chosen so that this guarantee is provided, there will be no conflict of interest, since agent A cannot dispossess agent B by his choice of asset. Consequently, he will choose the more valuable asset, asset 1. The required guarantee can be provided by imposing an additional incentive compatibility constraint on agent A's optimization problem:

\[
\tilde{U}_B (y(X) - P + W) dF_1 (X) = \tilde{U}_B (y(X) - P + W) dF_2 (X)
\]

(7)

where \( F_1 \) and \( F_2 \) are the cumulative distributions of the returns to assets 1 and 2 respectively.

In this environment, the optimal risk sharing contract, denoted \( \hat{y} \), will solve the optimization problem in (1), modified by replacing \( F(.) \) by \( F_1(.) \), and subject to the additional incentive compatibility constraint. If we substitute the incentive compatibility Constraint (7) into B's Individual Rationality Constraint (1b), agent A's problem can be written as:

\[
\max_y Z_A (X - y(X) - P) dF_1 (X) = Z_B (W + y(X) - P) dF_2 (X) = u_B
\]

The resulting risk sharing contract is described by the following proposition.
Proposition 4: The optimal solution to the asset owner's constrained problem is a contract which satisfies:

$$\frac{\partial \hat{y}}{\partial X} = \frac{R_A}{R_A + R_B} + \left( \frac{1}{R_A + R_B} \right) \left( \frac{f_1'}{f_1} - \frac{f_2'}{f_2} \right)$$

where $R_i$ is the Arrow-Pratt measure of absolute risk aversion, $f_i$ is the probability density function (p.d.f.) associated with $F_i$ for $i = 1, 2$ and $f_i'$ is the first derivative of the p.d.f. for each $i = 1, 2$.

Contract $\hat{y}$ provides agent A with the incentive to select the lower variance asset by providing agent A with a share in the selected asset's payoffs in bankruptcy states, as well as in non-bankruptcy states. This can be seen by noting that the first order condition for Contract $\hat{y}$ (as given by equation (9)) differs from the corresponding condition for the standard debt contract by the term $(f_1'/f_1 - f_2'/f_2)(R_A + R_B)^{-1}$. Since the standard deviations of the returns on the two assets differ, their p.d.f.'s also differ over the entire range of the asset's payoffs, causing this term to be non-zero over the entire range of payoffs. Whereas the standard debt contract provides a constant payment to the risk averse agent in the appropriate states and thereby transfers risk completely (to agent A in the good states and to agent B in the bad states), Contract $\hat{y}$ requires that neither agent receive a constant payment. This leads us to the following conclusion:

Proposition 5: If the agents' utility functions are given by equation (5), the optimal contract involves risk sharing between the two agents in all states.
The optimal contract in the presence of moral hazard is shown in Figure 2. In contrast to the standard debt contract in which agent A receives nothing in bankruptcy states, under Contract $\hat{y}$ agent A shares in the asset's returns in those states as well. In fact, as Figure 2 shows, agent A's payoff in the bankruptcy states is increasing in the asset's returns. As a consequence, under Contract $\hat{y}$ agent A is concerned with the selected asset's performance in bankruptcy states. It is this concern which ensures that the more valuable asset, asset 1, is chosen even when agent A has the opportunity to substitute assets.

III. MARKET FAILURE AND THE COURTS

In this section, we allow agents to trade contingent claims; while type A agents still make their asset choices through the firm's manager, all agents may, in addition, trade in derivative securities with payoffs contingent on the firm's cash flows. We also explicitly allow for many type A and type B agents.

A. Market Failure

To see why market failure occurs with Contract $\hat{y}$, let us assume that the manager acts to maximize shareholder wealth and issues Contract $\hat{y}$ in order to avoid the moral hazard problem. In a competitive environment where each agent takes the behavior of others as given, potential lenders may initially assume that the manager will remain with the lower variance project, asset 1--that is, that asset substitution will not occur.
However, once assured that asset 1 will be selected, both type A and B agents will find that they have an incentive to alter the payoff pattern that each receives from Contract $y$. That is, each will attempt to trade away from their contract obligations.

Consider a type B agent's behavior when the selection of the lower variance asset is taken as given. Under this condition, he would prefer the standard debt contract, which provides a non-random payment in the non-bankruptcy states (when he is risk averse) and an increasing payoff in the bankruptcy states (when he is risk neutral). Similarly, a type A agent would prefer the payoff pattern he would receive from the equity side of a standard debt contract. The question is: can these two agents engage in trades which provide them with their desired payoff patterns? If they are permitted to trade in derivative securities, the answer is yes. Consider the set of payoffs which are equal to the difference between those received by a type B agent under Contract $y$ and those received under the standard debt contract. These residual payoffs are positive in some states and negative in others. All that the type B agent needs to do to achieve a pattern of payoffs similar to those of the standard debt contract is to sell contingent claims yielding the positive residual payoffs to the type A agent and to buy from him, claims yielding the negative ones. This has the exact effect of bringing the type A agent from Contract $y$ to the standard debt contract. Hence the type A agent would be willing to sell what the type B agent wants to buy and vice versa. Of course, all agents would behave in a similar fashion. Consequently, even if managers issued only securities of the Contract $y$ type, all shareholders and lenders would end up holding, after trading on their own accounts, the payoff structures associated with the standard debt contract.

Now consider the behavior of a firm's manager under these conditions. The manager understands that when he issues Contract $y$, the firm's owners will trade in
derivative securities so as to ultimately hold securities with the same payoff pattern as that of equity levered by straight debt. He would realize that there is, once again, an opportunity to transfer wealth from the debtholders to the equityholders. Hence, following the stricture to maximize owners' wealth, managers would in fact substitute the higher risk asset for the lower risk asset. Therefore, Contract $\hat{y}$ is not a (Nash) equilibrium contract.

B. The Courts' Solution

The ability of agents to buy and sell derivative securities, and so "free ride" on the market, makes it impossible for firms' managers to eliminate the agency costs associated with the moral hazard problem. Even though Contract $\hat{y}$ makes all agents better off, it cannot be sustained in the market. However, an outside institution could make the market participants better off through the imposition of another Pareto-improving contract akin to Contract $\hat{y}$. One such institution is the legal system. Of course, the courts can only impose their contract at those times when they have jurisdiction over the parties, that is, in bankruptcy states. Consequently, in non-bankruptcy states, the payoffs from the court-imposed contract would have to be the same as those supported by the market. Those payoffs, as we saw in the previous section, are the payoffs of the standard debt contract.

The problem the courts face, then, is to find an optimal contract which pays debtholders a constant amount equal to $D$ in all non-bankruptcy states and which, in addition, eliminates the incentive for equity holders to engage in asset substitution. This problem is identical to the constrained owner's problem with the additional constraint that the
contract provide a constant payment equal to $D$ in the non-bankruptcy states. The court's problem can then be stated as:

$$\text{Max} \int U_A(X - y + P)dF_1(X)$$

subject to agent B's incentive compatibility constraint,

$$\int U_B(W + y(X) - P)dF_2(X) = u_B$$

the Market Feasibility requirement:

$$y(X) = D \quad \text{for all } X > D.$$  

In bankruptcy states, the solution to this problem (Contract $g$) satisfies:

$$\frac{\partial g}{\partial X} = \frac{R_A}{R_A + R_B} + \left( \frac{1}{R_A + R_B} \right) \left( \frac{f_1'}{f_1} - \frac{f_2'}{f_2} \right)$$

as in Proposition 4. Substituting $R_B = 0$ when $X \leq D$, we obtain:

$$\frac{\partial g}{\partial X} = 1 + \left( \frac{1}{R_A} \right) \left( \frac{f_1'}{f_1} - \frac{f_2'}{f_2} \right)$$

In non-bankruptcy states, the court's contract provides that agent B receive a constant payment $D$, as required by (12). We can now state:

**Proposition 6:** The optimal solution to the court's problem requires that equity holders share the risk of asset 1 in bankruptcy.

As shown in Figure 3, the payoff to both debt holders and equity holders is increasing in the asset's payoff for $X \leq D$. Debt holders get exactly $D$ whenever $X > D$, while equity holders get the residual. Again, it is the risk sharing in the bankruptcy states which ensures that the manager will choose the lower variance asset. Thus we see that
the moral hazard problem is resolved when the courts impose their own contract awarding equity a portion of the value of the firm in bankrupt states.

The entire argument for the efficiency of deviations from absolute priority can now be summarized. Agents' preferences lead the firm to issue standard debt. However, because of managers' options to switch to riskier assets, agency problems arise. Managers, knowing that any attempt to solve the agency problem through market mechanisms will only lead to offsetting actions by stock holders and bond holders, refrain from altering the form of the standard debt contract. All market participants realize, however, that if the firm does go into bankruptcy, the courts will deviate from absolute priority and award a positive share of the firm's assets to equity holders. Consequently the desired risk sharing occurs and perverse incentives for managers to switch to riskier assets are eliminated. The expected court award to stock holders will, of course, be taken into account when the market prices debt, but on the other hand, so will the elimination of agency costs.

C. Robustness Issues Regarding the Court's Solution

At this point, the objection may be raised that the solution imposed by the court is equally subject to being undone by equityholders and bondholders, and hence is not Nash either. However, on reflection, it can be seen that this is not true. The problem is not so much that contracts specifying a reassignment of payoffs under bankruptcy cannot be
written, but rather that they cannot be legally enforced. If the bankruptcy court is consciously deviating from absolute priority for social welfare reasons, it will clearly not enforce a contract that seeks to nullify the court's objectives. In practice, it would seem from the statements made by bankruptcy court judges, that they believe that they are not deviating from absolute priority. Consequently, it is not possible to get the court to enforce a contract that asks for the deviation from absolute priority to be reversed. Any enforcement of such an offsetting contract will require a valuation of cash flows different from the court's valuation and such an objective valuation clearly cannot be obtained from the court.\footnote{31}

A further objection that comes to the fore at this point regards the assumption that the issue of debt securities is induced by investors' preferences. Is this tantamount to assuming a special set of circumstances that generates the implications of the model regarding the impotence of capital structure to resolve agency problems? The answer, once again, is in the negative. As we have explained in footnote 21, the only assumption that is needed for our result is that investors prefer to hold some security other than strips;\footnote{32} and as explained above, this is really not a restriction on the result at all, since there would be no agency problems (of the type treated in this paper) at all if agents only wished to hold strips. In fact, we will argue that a condition for the existence of such agency problems is that the firm issue the non-strip security because investors have a preference for it (or that is, at least, one of the reasons). Formally, this can be stated as follows. Let V, W and Z stand for the following propositions:

V: "Investors do not have preferences for non-strip securities,"

W: "The firm issues non-strip securities," and

Z: "A moral hazard problem exists."

Then the assertion above can be rephrased as:
(W and Z) => ~ V

Logically, this is equivalent to the following assertion:

V => ~ (W and Z)

that is, V implies that W and Z cannot be simultaneously true. We now demonstrate this by considering two alternative at the corporate level reasons for the issuance by the firm of a non-strip security, say, straight debt: 1) corporate taxes and 2) signalling, and show that under these circumstances, no agency problem can exist.

Case 1: Assume that there are corporate taxes levied on the profits of the firm. Since debt service payments are tax-deductible, the manager will prefer to pay out some of the earnings of the firm as interest. Hence the firm will have some debt and some equity. Now, proposition V says that investors have no preferences for non-strip securities. Hence, all investor portfolios will consist of strips of the firm and there will be no incentive problems associated with debt.

Case 2: Now, assume that there is asymmetric information between corporate insiders and outsiders and this leads the manager to issue debt, which would otherwise not be issued. Again, the investors in the firm have no desire to hold the debt and the levered equity separately and hence they will end up holding strips.

In both these cases, the corporate capital structure contains debt; however, the ultimate portfolio held by the investors (including any home-made securities) will, in effect, not contain debt and hence there is no moral hazard problem. Hence the only way for the agency problems to exist is if investors have a preference for non-strip securities, such as debt. Since the investors themselves have no desire to hold non-strip securities, firm-level considerations will not affect their portfolio decisions.
IV. CONCLUSION

This paper provides a resolution to the problem of why bankruptcy courts award equity holders a share in the value of the firm even when that value is less than the sum of the contractual payments promised to debt holders. We show that an essential aspect of such deviation from absolute priority rules is that equity holders share in the value of the firm even in bad states. This value-sharing is anticipated even before bankruptcy and affects the manager's actions throughout the life of the firm. Consequently, the negative incentive effects of debt, as regards engagement in risk-seeking activity, are mitigated.

We show, furthermore, that the effectiveness of convertible debt to resolve this agency problem is greatly reduced if investors can trade in derivative securities with payoffs contingent on the cashflows of the firm. Under these circumstances, investors can undo the convertibility feature through trading on their own account; hence a manager desiring to maximize shareholder wealth will, once again, have an incentive to engage in excessive risk-taking.
APPENDIX

**Proposition 1:** The optimal solution to the asset owner's problem (1) satisfies:

\[
\frac{\partial y^*}{\partial X} = \frac{R_A}{R_A + R_B}
\]  

(2)

where \(R_i, i = A,B\), is the Arrow-Pratt measure of absolute risk aversion.

**Proof:** Following Rees (1985), the asset owner's problem can be solved through the pointwise maximization of the function \(h(y) = \{U_A + \lambda[U_B(y) - u_B]\}f(X)\) with respect to \(y\), where \(\lambda\) is the Lagrange multiplier and \(f(.)\) is the probability density function associated with \(F(.)\). As a result, the solution, \(y^*(X)\), specifying a payment from agent A to agent B is characterized by the following first order condition:

\[
-\left[ \frac{\partial U_A}{\partial y} (X - y^* + P) / \partial y \right] + \lambda \left[ \frac{\partial U_B}{\partial y} (y^* + W - P) / \partial y \right] = 0.
\]  

(3)

Equation (3) implies that the multiplier is equal to the ratio of the marginal utilities of income for the two agents.

Now differentiate the first order condition, equation (3), with respect to \(X\):

\[-\left( \frac{\partial^2 U_A}{\partial y^2} \right) (1 - \frac{\partial y^*}{\partial X}) + \lambda \left( \frac{\partial^2 U_B}{\partial y^2} \right) (\frac{\partial y^*}{\partial X}) = 0.\]

Finally, using the definition of the Arrow-Pratt measure of absolute risk aversion, and substituting for \(\lambda\) in terms of the marginal utilities from equation (3), we obtain equation (2) of the proposition. Q.E.D.

**Proposition 2:** If the agents' utility functions are given by (5), the standard debt contract is optimal.

**Proof:** The standard debt contract is given by

\[
y^*(X) = D \text{ for } X > D
\]

\[
y^*(X) = X \text{ for } X \leq D.
\]
We know from (5) that for $X > D$, $R_A = 0$. Hence, condition (2) becomes $\partial y^*/\partial X = 0$.

For $X \leq D$, $R_B = 0$. Hence condition (2) becomes $\partial y^*/\partial X = 1$. It is easy, now, to see that the standard debt contract satisfies the optimality condition (2). Q.E.D.

The next lemma will be used to prove Proposition 3, which follows.

**Lemma:** If $a_1$ and $a_2$ are normally distributed random variables with common mean $m$ and standard deviations $s_1$, $s_2$ respectively, with $s_1 < s_2$ and where $h(.)$ is an increasing, twice differentiable concave function, then $E[h(b_1)] < E[h(b_2)]$ where $b_1 = \max(0, a_1 - d)$ and $b_2 = \max(0, a_2 - d)$ for all finite $d$.

**Proof:** Since $a_1$ and $a_2$ are normally distributed with a common mean, it is true that
\[
\int_{-\infty}^{\infty} \left[ F_2(t) - F_1(t) \right] dt = 0
\]
where $F_1$ and $F_2$ are the cumulative density functions of $a_1$ and $a_2$ respectively. Further, since $s_1 < s_2$, it is also true that
\[
\int_{a}^{\infty} \left[ F_2(t) - F_1(t) \right] dt > 0, \text{ for all finite } a.
\]

From the definition of $b_1$ and $b_2$, it is easy to see that for all $t > 0$, $G_1(t) = F_1(t + d)$ and $G_2(t) = F_2(t + d)$, where $G_i$ is the cumulative distribution of $b_i$, $i = 1, 2$. Hence, equation (1) implies:
\[
\int_{0}^{\infty} \left[ G_2(t) - G_1(t) \right] dt < 0, \text{ and furthermore, for finite } a \geq 0,
\]
\[
\int_{a}^{\infty} \left[ G_2(t) - G_1(t) \right] dt < 0.
\]

Now, let $\overline{G}_1(t) = 1 - G_1(t)$ and $\overline{G}_2(t) = 1 - G_2(t)$. Then it follows from equation (2) that
\[
\int_{a}^{\infty} \overline{G}_2(t) dt > \int_{a}^{\infty} \overline{G}_1(t) dt.
\]
Proposition 8.5.1 in Ross (1983) then implies $E[h(b_1)] < E[h(b_2)]$. Q.E.D.

**Proposition 3:** Suppose the random payoffs of two assets are normally distributed with equal means and different variances. Then all risk averse individuals would prefer levered equity in the asset with the greater variance to that with the lower variance.

**Proof:** Let $a_1$ and $a_2$ be the payoffs from the two assets, where $a_1$ is distributed $N(\mu_1, \sigma_1)$ and $a_2$ is distributed $N(\mu_2, \sigma_2)$, with $\sigma_1 > \sigma_2$. Then $b_1 = \max(0,a_1-D)$ and $b_2 = \max(0,a_2-D)$ represent the payoffs to equity when the two assets are levered with equal amounts of debt with face value $D$. Let $U(.)$ represent the utility of an individual so that $U'' \leq 0$. Then, from the previous lemma, $E[U(b_1)] > E[U(b_2)]$. Q.E.D.

**Proposition 4:** The optimal solution to the asset owner's constrained problem is a contract which satisfies:

$$
\frac{\partial \tilde{y}}{\partial X} = \frac{R_A}{R_A + R_B} + \left( \frac{1}{R_A + R_B} \right) \left( \frac{f_1'}{f_1} - \frac{f_2'}{f_2} \right) \tag{9}
$$

where $R_i$ is the Arrow-Pratt measure of absolute risk aversion, $f_i$ is the probability density function (p.d.f.) associated with $F_i$ for $i = 1,2$ and $f_i'$ is the first derivative of the p.d.f. for each $i = 1,2$.

**Proof:** As in the proof of Proposition 1, the optimal solution to this problem can be solved by pointwise maximization of the function $g(y) = U_A f_2 + \lambda (U_B(y) - u_B) f_1$. Now, in a manner analogous to that of the previous proof, we can obtain equation (9). Q.E.D.
REFERENCES


FIGURE LEGENDS

Figure 1: Panel A: Standard debt: $y^*(\theta)$
Panel B: Standard equity: $x-y^*(\theta)$

Figure 2: Panel A: Incentive Compatible debt: $\$y(\theta)$
Panel B: Incentive Compatible equity: $x-\$y(\theta)$

Figure 3: Panel A: Debt Under the Court's Solution: $g(\theta)$
Panel B: Equity Under the Court's Solution: $x-g(\theta)$
Figure 1: Panel A: Standard debt: $y^*(\theta)$
Panel B: Standard equity: $x-y^*(\theta)$
Figure 2: Panel A: Incentive Compatible debt: $y^*(\theta)$
Panel B: Incentive Compatible equity: $x-y^*(\theta)$
Panel A

Figure 3: Panel A: Debt Under the Court's Solution: $y^*(\theta)$

Panel B

Figure 3: Panel B: Equity Under the Court's Solution: $x-y^*(\theta)$
We would like to thank David T. Brown, Ivan E. Brick, David Mauer and Anthony Tessitore for helpful comments as well as participants at the French Finance Association meetings, 1991, the Financial Management Association meetings, 1991, the ORSA-TIMS meetings, 1992, and workshops at the Rutgers University Graduate School of Management and the George Mason Center for Law and Economics. We also thank Carter Daniel for editorial assistance.


2 Another solution to this problem involves the use of bond covenants. However, the enforcement of bond covenants for such purposes would require a lot of monitoring and is likely to be costly. In fact, M.W. McDaniel, "Bondholders and Corporate Governance," 41 The Business Lawyer, 413 (1986) says that covenants restricting investments and disposition of assets are rare; and further that the trend is toward the abandonment of covenants in bond indentures.


4 Our argument applies more generally to all attempts to eliminate agency costs through capital structure. However, we restrict our discussion to convertible securities and the like, because the cost of undoing such capital structure effects seem to be much smaller than the cost of, say, creating homemade leverage.
5 For example, the popular graduate level textbook of T. E. Copeland and J. F. Weston Financial Theory and Corporate Policy at 478 (Reading, MA: Addison-Wesley Publishing Co., 3rd ed. 1988) says without any reservations: 'Green (1984) shows that agency costs between equity and bondholders are reduced by issuing convertible debt or straight debt with warrants.'


7 For example, J.C. Ritchie, "Convertible Securities and Warrants," in Frank J. Fabozzi and Irving M. Pollack (eds.) The Handbook of Fixed Income Securities, 2nd. ed., (1987), at 450 notes that smaller and more speculative firms often issue convertible bonds. He explains this in the following manner: The risks inherent in such issues tend to make it difficult to sell straight bonds or common stock at a reasonable cost. Management sweetens the debt issue by giving purchasers a chance to participate in
potential profits, while having a priority over equity securityholders in the event of financial difficulty. Similarly, Brigham (1966) reports that some CFOs used convertible debt because although their company wanted to issue straight debt, it found conditions to be such that a straight bond issue could not be sold at a reasonable rate of interest.


9 Elsewhere, he states: "the ultimate decision in many lawsuits is what allocation of resources would maximize efficiency. The market normally decides this question, but it is given to the legal system to decide in situations where the costs of a market determination would exceed those of a legal determination. The criteria of decision are often the same."


11 There are a whole host of such provisions in the bankruptcy code (see Barry E. Adler, "Bankruptcy and Risk Allocation", 77 *Cornell Law Journal* 155 (1992) ). For example, management has an exclusive first right for at least four months to propose a reorganization plan. Another example is provided by the bankruptcy code's cram down procedure (see Mark J. Roe, "Bankruptcy and Debt: A New Model for Corporate Reorganization", 83 *Columbia Law Review*, 527 (1983) for details). This is the procedure followed by the court to allocate the assets of the bankrupt firm when none of the proposed reorganization plans succeed. Under this procedure the court is supposed to follow a strict priority rule. However, in practice, the outcome of cram down favors
junior claimants by granting secured creditors securities with a nominal principal value equal to the adjudicated value of the firm's assets. Often a very low discount rate is used to price these securities, so that the secured creditors ultimately get less than their share under absolute priority. Consequently, the residual claimants, i.e. equity holders, are left with positive value.

12 Jackson and Scott, "On the Nature of Bankruptcy", Virginia Law Review, 75 (1989) 155 also suggest that court imposed deviations from absolute priority provide for risk-sharing between debtors and creditors to reduce agency costs. However, they are unable to show why the parties should not accomplish such risk-sharing by contractual means (Mark J. Roe, "Commentary on 'On the Nature of Bankruptcy': Bankruptcy, Priority and Economics", Virginia Law Review, 75 (1989) 219).

R.M. Giammarino, "The Resolution of Financial Distress," 2 The Review of Financial Studies, 25 (1989) argues that such violations arise as a result of recontracting by owners and creditors. Given the large deviations which are consistently observed, Giammarino's theory implies that the costs to creditors of enforcing their contractual rights are sizable. However, the paper provides no argument to expect that such costs are substantial. Moreover, Giammarino's model essentially ignores the existence of a complex body of legal procedures provided under Chapter XI of the Bankruptcy Reform Act of 1978 and the way in which these procedures are implemented.

D.T. Brown, "Claimholder Incentive Conflicts in Reorganization: The Role of Bankruptcy Law", 2 The Review of Financial Studies, 109 (1989) suggests that these violations are an attempt to increase efficiency in bargaining. Brown's approach explicitly incorporates bankruptcy laws as an interventionist assignment of rights to one
of the parties in the bankruptcy so as to reduce the probability of litigation and the consequent incurring of dissipative costs. Brown does not, however, explain why the process should favor equity, as it does, rather than another class of claim holder.


16 Eberhart, Moore and Roenfeldt, supra note 15.


18 Although the generation of debt through utility functions follows in the Arrow-Debreu tradition, it is nevertheless unusual in finance. However, we will show later in section III, that the more usual 'finance' reasons for the issuance of debt, such as signalling and taxes are incompatible with the existence of agency problems.

19 If it were indeed their most preferred payoff pattern, then convertible debt would have been issued in the first place, and there would be no agency problem. We assume however, as do Jensen and Meckling, that in fact the first best solution induces agency costs.
In practice, this may be done by selling traded options.

For convenience of exposition we assumed such utility functions as would have our investors hold either debt or equity, but this is not necessary for our results. As long as investors do not hold strips, there will be an incentive problem. If the manager attempts to resolve this incentive problem by forcing the investors to hold certain securities, they will undo his efforts by rearranging their portfolios. Hence the only assumption that our model requires is that investors have preferences such that they hold something other than strips.

In a two date world, as we will see, the only issue which arises is one of risk sharing between the agents. Our formulation is similar to that studied by S.A. Ross "The Economic Theory of Agency: The Principal's Problem", 63 American Economic Review (1973) 134 and "On the Economic Theory of Agency and the Problem of Similarity", in Balch, McFadden and Wu (eds.), Essays in Economic Behavior Under Uncertainty, Amsterdam North Holland Publishing Co., 1974.


That is, a bond (exogenous to the firm) with a face value of P. Henceforth, 'payment' will always mean payment in such bonds. We also assume that $W > P$ as a convenience to avoid dealing with fractions of securities.
25 All proofs can be found in the appendix.

26 Such a scenario might obtain, for example, if agent A had high exogenously endowed wealth in good states and agent B had high exogenously endowed wealth in bad states. Assuming that both agents have decreasing absolute risk aversion, their utility functions would be similar to those defined above.

27 Although we do not obtain an explicit valuation function for this economy, it can be shown (See Ingersoll, J.E., Jr., Theory of Financial Decision Making, Totowa, N.J.: Rowman & Littlefield, 1987 for example) that if payoffs are distributed normally and agents have risk averse utility functions, their utilities are increasing in the mean and decreasing in the variance. The definition of our utility functions guarantees that asset 1 is more valuable.

28 \[ P_2 \text{ satisfies } \int U_B (y^*(X) - P_2 + W) dF_2 = u_B. \]

29 Strictly speaking, equation (9) allows the payoff to the debt holder to be decreasing over some range in the return to the asset. If, for exogenous considerations, this is deemed undesirable, an alternative contract can be constructed, which has the same incentive properties as \( \hat{y} \), but is non-decreasing in \( X \). It can be shown that such a contract will also satisfy Proposition 4. For expositional purposes, we have chosen to depict such an alternative version of \( \hat{y} \) in Figure 2.

30 As \( X \) approaches \( D \) from below, the payoff to the equity holder under contract \( g \) drops from a positive amount to zero. This may create an additional moral hazard problem, in that the manager now has an incentive to hasten bankruptcy. To avoid this, if
it is desired that the payoff to the equity holder should be everywhere non-decreasing in $X$, such a contract can be easily created, without losing the desirable incentive properties guaranteed by Proposition 6.

31 For example, one of the ways in which deviations from absolute priority occur is when the court overvalues securities awarded to debtholders in a reorganization by using an overly low discount rate; because the equityholder in a reorganization remains in possession, he receives equity with positive value, violating absolute priority (see Brown (1989). Now in order to offset the deviations from absolute priority, we need to have trades in state contingent securities which require the equityholder to payback to the debtholder the difference between the 'true' value of the securities awarded to the debtholder in the reorganization and the value assigned to them by the court. But there is no objective way to determine the 'true' value of these securities. As far as the court is concerned the value it has assigned them is the true value. Hence the equity holder will not be able to enforce these offsetting contracts.

32 This demand for non-strip securities may be generated by a variety of causes; for example, personal taxes, risk attitudes or non-tradeable endowments. In all cases, other than where the demand is generated by risk attitudes, the utility functions referred to in section II may be considered as being derived utilities conditional on endowments, taxes and so on.